Mathematical Methods for Computer Science II

Spring 2021

Series 12 – Hand in before Monday, 31.05.2021 - 12.00

1. Construct a context-free grammar generating the language $(0 + 10)^*(1 + \epsilon)$.

2. Show that the following languages are not context-free.
   a) $\{a^i b^j c^k \mid i < j < k\}$
   b) $\{0^i \mid i = k^2 \text{ is a perfect square}\}$

3. Is the language $\{0^i 1^j 0^{i+j} : i, j \geq 1\}$
   a) regular?
   b) context-free?

4. Construct a PDA accepting the language of all balanced bracket expressions, that is, the language

$$L = \{w \in \{0, 1\}^* \mid \ell_0(w) = \ell_1(w), \ell_0(u) \geq \ell_1(u) \text{ whenever } w = uv\}.$$

   (For better readability, we replaced the left and the right brackets by 0s and 1s, respectively).

5. Let L be the set of all binary words containing an equal number of 0s as 1s.
   a) Construct a PDA accepting L by empty stack.
   b) Construct a PDA accepting L by final states.