Mathematical Methods for Computer Science II

Spring 2021

Series 11 – Hand in before Tuesday, 25.05.2021 - 12.00

1. What languages are generated by the following grammars? Explain your answer.
   a) $V = \{S\}, \quad T = \{(,)\}, \quad P = S \to (S) \mid SS \mid \varepsilon$
   b) $V = \{S\}, \quad T = \{(,), a, *\}, \quad P = S \to (S * S) \mid a$

2. a) Give a context-free grammar generating the language of all predicate formulas with variables p, q and logical connectives $\neg, \to$. Each subformula $F \to G$ must be surrounded by parentheses: $(F \to G)$.
   b) Let G be a grammar in Chomsky form, and let $w \in L(G)$ be a word of length n. How long is a derivation of w in G? (That is, how many direct derivations does one need to derive w from S?)

3. a) Give a context-free grammar generating the set of all non-empty palindromes (words that read the same forward and backward) over the alphabet $\{a, b\}$.
   b) Give a Chomsky form context-free grammar generating the same language.

4. Eliminate the ε- and unit productions from the grammar
$$S \to ASB \mid c \quad A \to aAS \mid \varepsilon \quad B \to SBb \mid A \mid b.$$

5. Show that the ε- and unit productions must be eliminated in a correct order. That is, give an example of a grammar G such that if one first eliminates the unit, and then the ε-productions using our algorithms, then the resulting grammar contains unit productions.