Mathematical Methods for Computer Science II

Spring 2021

Series 10 – Hand in before Monday, 17.05.2021 - 12.00

1. Show that the following languages are not regular:
   a) The set of all binary words of odd length with 1 in the middle: $\{u1v \mid |u| = |v|\}$.
   b) The set of all "words repeated twice": $\{ww \mid w \in \{0,1\}^*\}$.

2. Let L be the language of all binary words without two consecutive zeros.
   a) Describe the equivalence classes of binary words with respect to L.
   b) Construct a minimal DFA accepting L and indicate which state of this DFA corresponds to which equivalence class under $\sim_L$.

3. Let $L \subset \Sigma^*$ be a language, and let $a \in \Sigma$ be a letter of the alphabet. The *quotient of L by a* is the following language:
   $$L/a = \{w \in \Sigma^* \mid wa \in L\}.$$
   a) Show that $L/a \cdot \{a\} \subset L$, but not necessarily $L/a \cdot \{a\} = L$. Here, $\cdot$ stands for the concatenation of languages.
   b) Show that $u \sim_L v \Rightarrow u \sim_{L/a} v$.
   c) Show that if L is a regular language, then the language L/a is also regular.

4. Minimize the DFA given by the following table.

	0	1
q_0	q_2	q_0
q_1	q_6	q_7
q_2	q_0	q_3
q_3	q_7	q_2
q_4	q_7	q_5
q_5	q_6	q_4
q_6	q_5	q_6
q_7	q_7	q_0

   Here, q_0 is the initial state, and q_7 is the only final state.

5. Let L be the language of all binary words of length ≥ 10 whose tenth symbol from the right is 1. What is the minimum number of states in a DFA accepting L?