Mathematical Methods for Computer Science II

Spring 2021

Series 9 – Hand in before Monday, 10.05.2021 - 12.00

1. For a binary word w denote by $\ell_0(w)$ the number of 0's in w, and by $\ell_1(w)$ the number of 1's in w. Which of the following languages are regular? Justify your answer.
   a) $\{w \in \{0,1\}^* \mid \ell_0(w) > \ell_1(w)\}$
   b) $\{w \in \{0,1\}^* \mid \ell_0(w) \neq \ell_1(w)\}$

2. Which of the following languages are regular? Justify your answer.
   a) All binary palindromes (words which read the same backwards as forwards).
   b) All binary words of length at least two whose first symbol is the same as the second symbol from the end.

3. a) Prove the following extension of the pumping lemma. Let L be a regular language. Then there is a positive integer n such that for any words z_1, z_2, z_3 whose concatenation $z_1 z_2 z_3$ is in L and $|z_2| \geq n$ the middle word z_2 can be written as $z_2 = uvw$ with $|v| \geq 1$ so that $z_1 uv^k wz_3 \in L$ for all $i \geq 0$.
   b) Prove that the language $\{a^i b^j c^j \mid i \geq 50, j \geq 0\}$ is not regular.

4. Let $\Sigma = \{a,b,c\}$ and $\Delta = \{0,1\}$. Consider the homomorphism $h\colon \Sigma^* \to \Delta^*$ defined by $h(a) = 0$, $h(b) = 01$, $h(c) = 10$.
   a) Describe the homomorphic image $h(\Sigma^*)$ in a human language.
   b) Let $L \subset \Delta^*$ be the language described by the regular expression $0(10)^*$. Find a regular expression for $h^{-1}(L)$ and justify your answer.

5. Let $\operatorname{rev}\colon \Sigma^* \to \Sigma^*$ be a map that reverses the words:
$$\operatorname{rev}(a_1 \ldots a_n) = a_n \ldots a_1.$$
   Show that if $L \subset \Sigma^*$ is a regular language, then so is $\operatorname{rev}(L)$.