## Mathematical Methods for Computer Science II

Spring 2021

Series 8 - Hand in before Monday, 03.05.2021-12.00

1. Let $M_{i}, i=1,2,3$, be $\varepsilon$-NFAs with a unique final state, and $L_{i}$ be the language accepted by $M_{i}$. Using the algorithm given in the lecture, sketch an $\varepsilon$-NFA that accepts the language $\left(L_{1} \cup L_{2}\right) L_{3}$ and an automaton that accepts the language $\left(L_{1} L_{3}\right) \cup\left(L_{2} L_{3}\right)$.
2. Let $M$ be an $\varepsilon$-NFA with $n$ states accepting a language $L$. Sketch an $\varepsilon$-NFA $M^{\prime}$ accepting the language $L\left(L^{*}\right)$ such that its number of states is also $n$. (In $M^{\prime}$ transitions from the final state are allowed.)
3. Let $\left(Q_{i}, \Sigma, \delta_{i}, q_{i}, F_{i}\right), i=1,2$, be two DFAs accepting the languages $L_{1}$ and $L_{2}$, respectively. Let $n_{1}$ and $n_{2}$ be the number of states in the first and in the second automaton, respectively. Describe a DFA with $n_{1} n_{2}$ states that accepts the language $L_{1} \cup L_{2}$ and a DFA that accepts the language $L_{1} \cap L_{2}$. In both cases, give a formal description of the set of states, of the transition function, and of the set of final states.
4. Find a regular expression for the language accepted by the automaton shown below.

5. Construct a regular expression for the language of all binary words with an even number of zeros and an even number of ones.
