## Mathematical Methods for Computer Science II

Spring 2021

Series 7 - Hand in before Monday, 26.04.2021-12.00

1. Construct a DFA equivalent to the $\varepsilon$-NFA given by the following table.

|  | $\varepsilon$ | $a$ | $b$ | $c$ |
| :---: | :---: | :---: | :---: | :---: |
| $q_{0}$ | $\left\{q_{1}, q_{2}\right\}$ | $\varnothing$ | $\left\{q_{1}\right\}$ | $\left\{q_{2}\right\}$ |
| $q_{1}$ | $\varnothing$ | $\left\{q_{0}\right\}$ | $\left\{q_{2}\right\}$ | $\left\{q_{0}, q_{1}\right\}$ |
| $q_{2}$ | $\varnothing$ | $\varnothing$ | $\varnothing$ | $\varnothing$ |

The initial state is $q_{0}$, the only final state is $q_{2}$.
2. a) Construct an $\varepsilon$-NFA for the set of binary words consisting either of repeating 01 or of repeating 010 (the empty word $\varepsilon$ also belongs to this language).
b) Construct a finite automaton (DFA, or NFA, or $\varepsilon$-NFA) for the set of binary words which contain at least one symbol 1 in the last three positions. Try to minimize your construction.
3. Describe the following languages in a human language as briefly as possible.
a) $\left(0^{*} 1^{*}\right)^{*}$
b) $(0+1)^{*} 0(0+1)^{*} 0(0+1)^{*}$
c) $\left(1^{*} 01^{*} 01^{*}\right)^{*}$
d) $(1+01)^{*}$
4. Find regular expressions for the following languages.
a) Sequences of zeros of length $n$ such that $n$ dollars can be changed into bills of 17 and 31 dollars.
b) Binary words with two consecutive 1s at some place.
c) Binary words without two consecutive 1s.
d) Binary words whose number of 0s is odd.
5. Show that for every $\varepsilon$-NFA with $n$ states and at least one $\varepsilon$-transition there is an equivalent DFA with at most $3 \cdot 2^{n-2}$ states.

