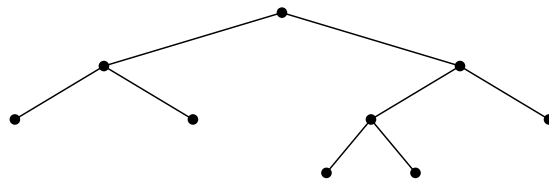


Mathematical Methods for Computer Science II

Spring 2021

Series 5 – Hand in before Monday, 12.04.2021 - 12.00

1. For the full binary tree shown below, give the corresponding bracket-variable expression and the corresponding triangulation of a polygon (mark the base edge of the polygon).



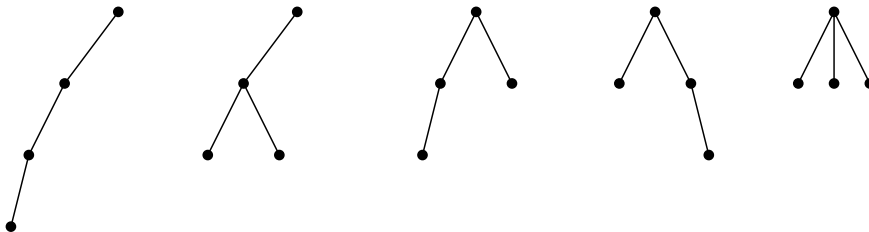
2. a) Using the formula for c_n prove that

$$(4n + 2)c_n = (n + 2)c_{n+1}.$$

- b) Prove the above relation between c_n and c_{n+1} with the help of the following picture. (This can be used to give an alternative proof of $c_n = \frac{1}{n+1} \binom{2n}{n}$.)



3. Show that the number of rooted trees with n edges is equal to the n -th Catalan number. This time, the degrees of vertices (including the root) are arbitrary. The picture below shows all rooted trees with 3 edges. (Hint: find a bijection to the set



of Dyck paths from $(0, 0)$ to (n, n) .)

4. In how many different ways can $2n$ people at a round table shake hands so that no arms are crossed and everybody gives a handshake?
5. Show that the Catalan number c_n is odd if and only if $n = 2^k - 1$ for some k .