UNIVERSITÉ DE FRIBOURG — DÉPARTEMENT DE MATHÉMATIQUES
UNIVERSITÄT FREIBURG — MATHEMATIK DEPARTEMENT

Mathematical Methods for Computer Science II

Spring 2021

Series 4 – Hand in before Monday, 29.03.2021 - 12.00

1. Describe the combinatorial meaning of the coefficient at x^n in the formal power series
$$\prod_{k=0}^{\infty}(1+x^{2k+1}).$$

2. Denote by $p(n,k)$ the number of partitions of n into *exactly* k parts, and by $p(n,\leq k)$ the number of partitions of n into *at most* k parts.
   a) Show that $p(n-k,\leq k)=p(n,k)$.
   b) Show that $p(n,k)=p(n-1,k-1)+p(n-k,k)$.

3. Show that
$$\sum_{n=0}^{\infty}p(n,\leq k)x^n=\frac{1}{(1-x)(1-x^2)\cdots(1-x^k)},$$
$$\sum_{n=0}^{\infty}p(n,k)x^n=\frac{x^k}{(1-x)(1-x^2)\cdots(1-x^k)}.$$

4. Show that the number of all partitions of n is even if and only if the number of partitions of n into distinct odd parts is even.

5. The *Durfee square* is the largest square that fits into the Ferrers diagram of a partition, see Figure below. Using the Durfee square (and a theorem from the course) show that the number of partitions of n into distinct odd parts is equal to
$$\sum_{k=1}^{\infty}p\left(\frac{n-k^2}{2},\leq k\right).$$
(If $\frac{n-k^2}{2}$ is not an integer, then the corresponding term is put to be zero. In other words, the sum goes over even k if n is even and over odd k if n is odd.)