Université de Fribourg  Département de Mathématiques
Universität Freiburg  Mathematik Departement

# Mathematical Methods for Computer Science II

Spring 2021

Series 2 – Hand in before Monday, 15.03.2021 - 12.00

1. Find the coefficients $a_0$, $a_1$, $a_2$, $a_3$ of the formal power series
$$\sum_{k=0}^{\infty} a_k x^k = \frac{1}{1-x+2x^3}$$
   a) by solving a system of linear equations arising from
$$(1-x+2x^3)(a_0+a_1x+a_2x^2+a_3x^3+\cdots)=1,$$
   b) by performing an appropriate substitution into
$$\frac{1}{1-y} = 1+y+y^2+\cdots$$

2. Let $(a_0, a_1, a_2, \ldots)$ be a sequence satisfying the recursive relation
$$a_n = r_1 a_{n-1} + r_2 a_{n-2} + \cdots + r_k a_{n-k}, \quad n \geq k.$$
   Show that its generating function $A(x) = a_0 + a_1 x + a_2 x^2 + \cdots$ has the form
$$A(x) = \frac{B(x)}{\overline{P}(x)}, \quad \overline{P}(x) = 1 - r_1 x - r_2 x^2 - \cdots - r_k x^k.$$

3. Write the generating function of the sequence
$$a_0 = -1, \quad a_1 = 1, \quad a_2 = 2, \quad a_n = 3a_{n-2} - 2a_{n-3}$$
   as the quotient of two polynomials.

4. Show that
$$\sum_{k=0}^{\infty} k^2 x^k = \frac{1}{1-x} - \frac{3}{(1-x)^2} + \frac{2}{(1-x)^3}.$$

5. a) Find a closed form expression for the generating function of the sequence
$$a_0 = 1, \quad a_{n+1} = 2a_n + 3$$
   b) Find a closed form expression for the terms of the above sequence.