Université de Fribourg | Département de Mathématiques
Universität Freiburg | Mathematik Departement

Mathematical Methods for Computer Science II

Spring 2021

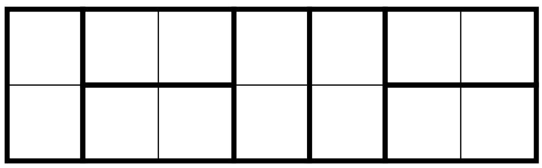
Series 1 – Hand in before Monday, 08.03.2021 - 12.00

1. a) Expand $(1+\sqrt{5})^n$ and $(1-\sqrt{5})^n$ using the binomial theorem.
   b) Prove the following formula for Fibonacci numbers:
$$a_n = \frac{\binom{n}{1} + 5\binom{n}{3} + 5^2\binom{n}{5} + \cdots}{2^{n-1}}.$$

2. Derive a closed formula for the n-th term of the recursive sequence
$$a_0 = 0, \quad a_1 = 1, \quad a_n = 5a_{n-1} - 6a_{n-2}.$$

3. Derive a closed formula for the n-th term of the recursive sequence
$$a_0 = -1, \quad a_1 = 1, \quad a_2 = 2, \quad a_n = 3a_{n-2} - 2a_{n-3}.$$

4. a) Let b_n be the number tilings of the rectangle of size $2 \times n$ with dominoes. The figure below shows an example of such a tiling for $n = 7$. Show that $b_n = a_{n+1}$, where a_n is the Fibonacci sequence: $a_1 = a_2 = 1$, $a_{n+1} = a_n + a_{n-1}$.



   b) How many perfect matchings does the graph $P_{2,n}$ have? (Here, $P_{m,n}$ denotes the graph of the square grid on $m \times n$ vertices.)

5. Let a_n denote the sequence of Fibonacci numbers:
$$a_1 = 1, \quad a_2 = 1, \quad a_n = a_{n-1} + a_{n-2} \text{ for } n > 2.$$
   a) Prove that for any $1 \leq k \leq n-2$, we have the formula
$$a_n = a_{k+1}a_{n-k} + a_k a_{n-k-1}.$$
   b) Show that if m is divisible by n, then a_m is divisible by a_n.