# Mathematical Methods for Computer Science I

Fall 2020

Series 10 – Hand in before Monday, 30.11.2020 - 12.00

1. Construct a proof tree or a deduction tree with a counterexample leaf for each of the following propositional formulas:
   a) $p \to (q \to (p \wedge q))$
   b) $(p \vee q) \to ((p \to q) \vee q)$

2. Construct closed deduction trees and use them to find a conjunctive normal form for the following formulas:
   a) $(p \to r) \to ((q \to s) \to ((p \vee q) \to r))$
   b) $(p \to q) \to ((q \to \neg r) \to \neg p)$

3. Find Gentzen-like inference rules for the connective $\leftrightarrow$ defined as
$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$
   That is, find inference rules eliminating $\leftrightarrow$ from the precedent and from the succedent:
$$\frac{}{\Gamma \vdash A \leftrightarrow B, \Delta} \quad \text{and} \quad \frac{}{\Gamma, A \leftrightarrow B \vdash \Delta}.$$

4. Explain how to construct a DNF for a proposition $A$ with the help of a closed deduction tree with the sequent $A \vdash$ as the root. Prove the correctness of your construction.

5. Consider closed deduction trees with the root $\vdash A$, where $A$ is some propositional formula. Recall that there are different closed deduction trees with the same root: when constructing a tree, at each vertex we have a choice from which formula to eliminate the top level logical connective.
   a) Assume that $A$ contains $m$ logical connectives. Show that the number of leaves in a closed deduction tree is at most $2^m$.
   b) Give an example of a formula with $2k - 1$ connectives whose closed tree has $2^k$ leaves.
   c) (*Bonus question*) Is the number of leaves the same in all closed deduction trees for a given formula?