## Mathematical Methods for Computer Science I

Fall 2020

Series 9 – Hand in before Monday, 23.11.2020 - 12.00

1. a) Show that the set of connectives $\{\neg, \rightarrow\}$ is complete (that is, every Boolean function can be expressed by a formula that contains only these connectives).
   b) The connective $\uparrow$ is defined as $p \uparrow q = \neg(p \wedge q)$. (Whence its other name NAND.) Show that this single connective suffices to express every Boolean function. In other words, show that the set of connectives $\{\uparrow\}$ is complete.

2. Write the formula $(p \rightarrow q) \wedge ((q \vee r) \rightarrow p)$ in a disjunctive normal form
   a) by transforming it with the help of the distributive laws, de Morgan laws and other logical equivalences;
   b) by filling in a truth table and reading its rows.

3. Sometimes a propositional formula in DNF can be simplified to an equivalent formula also in DNF.
   a) Transform the formula
   $$(\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge q \wedge r)$$
   to a simpler formula in DNF.
   b) Transform the formula
   $$(\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (p \vee q \vee r)$$
   to a simpler formula in CNF.

4. Show that a Hilbert proof system (a set of axioms plus the modus ponens inference rule) is sound if and only if all of its axioms are tautologies.

5. a) Show that a propositional formula in DNF can be satisfied unless every conjunctive clause contains a literal as well as its negation.
   b) Using distributivity, show that every formula in CNF can be transformed into an equivalent formula in DNF.
   c) The first part of this exercise can be used to give a very fast algorithm verifying whether a given propositional formula in DNF is satisfiable. Discuss why it might take much longer so verify whether a formula in CNF is satisfiable, even though every such formula is equivalent to a formula in DNF (for example via the procedure described the second part of this exercise).

   *Remark.* We do not deal with complexity theory in this course. Just in case you know the terms (or would like to learn them): satisfiability of a formula in DNF can be checked in linear time (with respect to the length of the formula), while checking satisfiability in CNF is known to be NP-complete.