Université de Fribourg
Universität Freiburg

Département de Mathématiques
Mathematik Departement

Mathematical Methods for Computer Science I

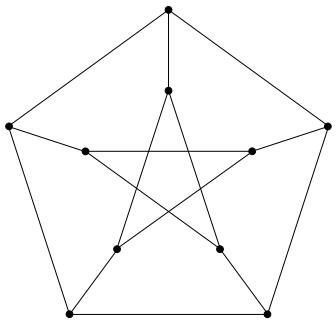
Fall 2020

Series 6 – Hand in before Monday, 02.11.2020 - 12.00

1. A planar graph G is called *maximal planar* if the addition of any edge to G creates a nonplanar graph.
   a) Show that every face of a maximal planar graph is a triangle.
   b) If a maximal planar graph has n vertices, how many edges and faces does it have? (*Hint*: Use the theorem on the sum of degrees of faces and the Euler formula.)

2. The *girth* of a graph is the length of a shortest cycle contained in the graph. Show that for every connected planar graph with girth k, we have
$$|E| \leq \frac{k(|V|-2)}{k-2}.$$

3. Prove the non-planarity of the Petersen graph in three different ways:
   a) using Exercise 3,
   b) by applying Kuratowski's theorem,
   c) by applying Wagner's theorem.



4. a) Show that for every 3-regular graph, it holds that
$$3|V| = 2|E|.$$
   b) Let G be a connected 3-regular plane graph with only pentagonal and hexagonal faces. Show that it has exactly 12 pentagonal faces.

5. Assume you are given pentagonal and hexagonal patches of fabric, all having equal side length. Sewing the patches together along their sides so that at every corner, two hexagonal patches and one pentagonal patch meet, you want to create a ball. Argue that the only way to do so is by using exactly 12 pentagonal and 20 hexagonal patches.