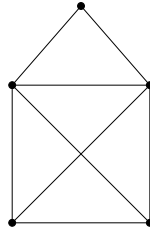

Mathematical Methods for Computer Science I

Fall 2020

Series 5 – Hand in before Monday, 26.10.2020 - 12.00

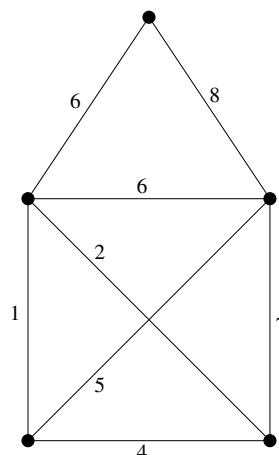
1. Consider the following “house graph” G on five vertices:



- a) Is G Eulerian? Is G semi-Eulerian?
b) Is there an Euler walk on G starting at the top of the roof (that is, at the vertex of degree two)? Give an example or prove that no such Euler walk can exist.
2. How many different (that is, pairwise non-isomorphic) trees on 1, 2, 3, 4, 5 or 6 vertices are there? Draw all of them.
3. a) Show that in every tree $T = (V, E)$, the following equation holds:

$$\sum_{v \in V} \deg v = 2|V| - 2.$$

- b) Assume that a historical Person A had 4 children, 10 of Person A’s descendants had 3 children each, 15 had 2 children, and all other descendants died childless. How many descendants did Person A have?
4. Assume that a tree T has a vertex of degree d . Show that T has at least d leaves (that is, vertices of degree one).
5. a) Apply Kruskal’s algorithm to find a minimum spanning tree in the weighted graph shown below.



- b) Give an example of two vertices in this graph such that the path between them in the minimum spanning tree is heavier than the path of minimum weight between them in the graph.