Mathematical Methods for Computer Science I

Fall 2020

Series 3 – Hand in before Monday, 12.10.2020 - 12.00

1. Prove
$$\binom{n}{k_1,\dots,k_r} = \binom{n}{k_r}\binom{n-k_r}{k_1,\dots,k_{r-1}}$$
   a) using the formula for multinomial coefficients;
   b) by a combinatorial argument.

2. a) Show that for $n = k + l + m$ we have
$$\binom{n}{k,l,m} = \binom{n-1}{k-1,l,m} + \binom{n-1}{k,l-1,m} + \binom{n-1}{k,l,m-1}.$$
   b) Show that for every n we have
$$\sum_{\substack{k+l+m=n\\ k,l,m\geq 0}} \binom{n}{k,l,m} = 3^n.$$

3. Of 33 students, 20 ski, 15 climb, 8 play ice hockey. (Some students might do none of these sports.) Besides, 6 ski and climb, 2 ski and play ice hockey, and 3 climb and play ice hockey. Show that there is at most one student who does all three sports.

4. This exercise deals with the number of surjective maps from an n-element set to an m-element set (we have not considered this problem so far).
   a) Calculate this number for $m = 2$.
   b) Calculate this number for $m = 3$.
   c) Write a formula for this number that holds for general m.
   (*Hint*: The resulting formula is given by a sum and not a “nice” formula like a binomial coefficient).

5. a) In the hats problem from the lecture, what is the probability that exactly one guest gets their hat?
   b) (A different hat problem.)
   The guests are leaving the party one after the other. The first guest takes a hat by chance. Each of the following guests looks for their hat. If they find their hat, then they take it; if not, then they take any hat by chance. What is the probability that the last guest gets their hat?