Mathematical Methods for Computer Science I

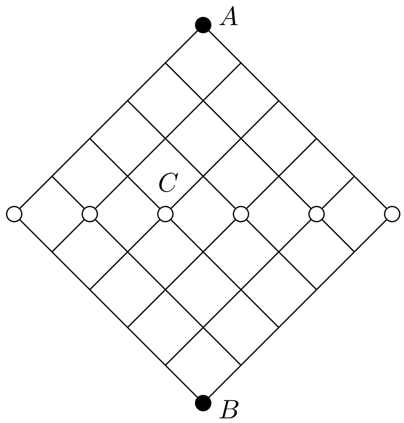
Fall 2020

Series 2 – Hand in before Monday, 05.10.2020 - 12.00

1. a) Let m, n, k be positive integers such that $k \leq m$ and $k \leq n$. Prove:
$$\binom{m+n}{k} = \binom{m}{0}\binom{n}{k} + \binom{m}{1}\binom{n}{k-1} + \cdots + \binom{m}{k}\binom{n}{0}.$$
(*Hint:* think of taking k balls from a bag with m black and n white balls. In how many ways do you get i black and $k-i$ white balls?)

   b) Prove the same identity by looking at the coefficient of x^k on the left hand side and on the right hand side of
$$(1+x)^{m+n} = (1+x)^m(1+x)^n.$$

2. a) How many lattice paths are there that go from A to B moving only downwards and passing through the point C?



   b) Prove the identity
$$\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$
using the fact that any path from A to B passes through one of the white points.

   c) Prove the same identity using the result of Problem 1.

3. Draw a convex n-gon and all of its diagonals. How many intersection points between the diagonals are there, if no three of them pass through a common point? (For example, in a quadrilateral there is one intersection point, and in a pentagon there are five.)

4. Prove the identity
$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \cdots + (-1)^k\binom{n}{k} = (-1)^k\binom{n-1}{k}$$
(Hint: use induction on k.)

5. Recall that a composition of a positive integer n is a representation of n as the sum of positive integers, taking into account the order of summands. The number of summands can range between 1 and n, in particular the compositions
$$n = n \quad \text{and} \quad n = \underbrace{1 + \cdots + 1}_{n}$$
are possible. How many different compositions of n are there?