

## JOURNÉES DE GÉOMÉTRIE HYPERBOLIQUE 2024 – PROGRAM

The talks of the **conference** will take place in **Room 2.52** of the **Physics Building PER 8**.

The **Public thesis presentation** takes place at **Pérolles PER21, G120**

<i>Physics 2.52</i>	<b>Thursday</b>	<i>Physics 2.52</i>	<b>Friday</b>
		<b>09:15 - 10:00</b>	<b>Jean Rimbault</b> <i>Coxeter groups in hyperbolic space, random and deterministic</i>
		<i>Coffee break</i>	<i>Mensa</i>
<b>10:30</b>	<i>Doctoral exam Naomi Bredon (not public)</i>	<b>10:30 - 11:15</b>	<b>Anna Felikson</b> <i>Polytopal realizations of non-crystallographic associahedra</i>
		<b>11:30 - 12:15</b>	<b>Livio Liechti</b> <i>Multicurve bipartite degrees</i>
		<i>lunch</i>	<i>Restaurant "Les menteurs"</i>
<b>14:00 - 14:45</b>	<b>Michelle Bucher</b> <i>Continuous cocycles on the Furstenberg boundary and applications to bounded cohomology</i>	<b>14:30 - 15:15</b>	<b>Yohei Komori</b> <i>On the growth rate of hyperbolic 4-dimensional compact Coxeter polytopes</i>
<i>Coffee break</i>	<i>Mensa</i>		
<b>15:15 - 16:00</b>	<b>Pavel Tumarkin</b> <i>Polytopal realizations of non-crystallographic associahedra</i>		
<b>16:15 - 17:00</b>	<b>Alain Valette</b> <i>Reciprocal hyperbolic elements in <math>PSL_2(\mathbb{Z})</math></i>	<b>16:30 - 17:30</b>	<i>Public thesis presentation, Pérolles II, G120:</i> <b>Naomi Bredon</b> <i>On the existence of hyperbolic Coxeter groups</i>
		<b>17:30</b>	<b>Apéro</b> <i>Pérolles II, G130</i>
<b>18:30</b>	<b>Conference Dinner</b> <i>Restaurant Aigle Noir</i>		

## ABSTRACTS

**Naomi BREDON** (Fribourg) :

### **On the existence of hyperbolic Coxeter groups**

A Coxeter polyhedron in a standard geometric space is a convex polyhedron of finite volume all of whose dihedral angles are integral submultiples of  $\pi$ . The group generated by the reflections in the facets of a Coxeter polyhedron is called a Coxeter group. In contrast to the spherical and Euclidean cases, Coxeter groups acting on hyperbolic spaces of dimensions beyond 3 are far from being classified. During my PhD, I studied two different aspects of hyperbolic Coxeter groups. In this presentation, I will focus on my contribution to the classification problem of hyperbolic Coxeter groups.

First I will provide a friendly introduction to the subject (in French). Then, I will give an overview of known classification results of hyperbolic Coxeter polyhedra and present the classification of a new family of polyhedra, the ADEG-polyhedra. This family consists of all polyhedra with mutually intersecting facets whose dihedral angles are  $\frac{\pi}{2}$ ,  $\frac{\pi}{3}$  and  $\frac{\pi}{6}$ . I will provide the main ingredients for the proof, together with some properties of the new Coxeter polyhedron  $P_*$  in  $\mathbb{H}^9$ .

**Michelle BUCHER** (Geneva) :

### **Continuous cocycles on the Furstenberg boundary and applications to bounded cohomology**

Group cohomology comes in many variations. The standard Eilenberg-MacLane group cohomology is the cohomology of the cocomplex

$$\{f : G^{q+1} \rightarrow \mathbb{R} \mid f \text{ is } G\text{-invariant}\}$$

endowed with its natural homogeneous coboundary operator. Now whenever a property  $P$  of such cochains is preserved under the coboundary one can obtain the corresponding  $P$ -group cohomology.  $P$  could be: continuous, measurable,  $L^0$ , bounded, alternating, etc. Sometimes these various cohomology groups are known to differ (eg  $P$ =empty and  $P$ =continuous for most topological groups), in other cases they are isomorphic (eg  $P$ =empty and  $P$ =alternating (easy),  $P$ =continuous and  $P=L^0$  (a highly nontrivial result by Austin and Moore valid for locally compact second countable groups)).

In 2006, Monod conjectured that for semisimple connected, finite center, Lie groups of noncompact type, the natural forgetful functor induces an isomorphism between continuous bounded cohomology and continuous cohomology (which is typically very wrong for discrete groups). I will focus here on the injectivity and show its validity in several new cases including isometry groups of hyperbolic  $n$ -spaces in degree 4, known previously only for  $n = 2$  by a tour de force due to Hartnick and Ott.

Monod recently proved that all such continuous (bounded) cohomology classes can be represented by measurable (bounded) cocycles on the Furstenberg boundary. Our main result is that these cocycles can be chosen to be continuous on a subset of full measure. In the real hyperbolic case, this subset of full measure is the set of distinct tuples of points, easily leading to the injectivity in degree 4.

This is joint work with Alessio Savini.

**Anna FELIKSON** (Durham) :

**Polytopal realizations of non-crystallographic associahedra** An associahedron is a polytope arising from combinatorics of Catalan-type objects (for example, from a collection of all triangulations of a given polygon). Fomin and Zelevinsky found a way to construct the same combinatorial structure by considering the Coxeter group of type  $A_n$ . This allowed them to define a generalized associahedron for every finite reflection group. For generalized associahedra arising from crystallographic reflection groups, it was also shown that they can be realized as polytopes. We use the folding technique to construct polytopal realisations of generalized associahedra for all non-simply-laced root systems, including non-crystallographic ones. This is a joint work with Pavel Tumarkin and Emine Yildirim.

I will sketch the history of the associahedron and introduce generalised associahedra, then we will discuss how to produce the associahedra in the non-crystallographic case. The talk will not require any special background.

**Yohei KOMORI** (Waseda) :

**On the growth rate of hyperbolic 4-dimensional compact Coxeter polytopes**

I will give a talk on two topics concerning the growth rate of hyperbolic 4-dimensional compact Coxeter polytopes. The first topic addresses the problem of enumerating hyperbolic 4-dimensional compact Coxeter polytopes with small growth rates. The second topic involves the investigation of hyperbolic 4-dimensional compact Coxeter polytopes whose growth rate is a 2-Salem number.

**Livio LIECHTI** (Fribourg) :

**Multicurve bipartite degrees**

A pair of multicurves that fill a closed orientable surface determines a bipartite graph whose vertices correspond to curve components and the number of edges between a pair of vertices equals the number of intersection points of the respective curve components. We discuss algebraic properties the spectral radii of adjacency matrices of such graphs. In particular, we prove that the maximal algebraic degree of such a spectral radius achieved on a surface of genus  $g > 1$  is exactly  $6g - 6$ . Our technique can be used to prove a claim on pseudo-Anosov stretch factors by Thurston from the 1980s. This talk is based on joint work with Erwan Lanneau.

**Jean RAIMBAULT** (Marseille) :

**Coxeter groups in hyperbolic space, random and deterministic** I will present two constructions of Coxeter groups in hyperbolic space with interesting properties : the first is a construction of compact non-arithmetic polyhedra in hyperbolic 4- and 5-space to Makarov in the 60's, and in a recent work with Bogachev and Douba we showed that they form infinitely many commensurability classes in both dimensions. The second is a general method to produce so-called "invariant random subgroups" in certain Coxeter groups, which yields some examples with interesting properties for some low-dimensional examples.

**Pavel TUMARKIN** (Durham) :

**Friezes associated to a pair of pants**

Frieze patterns (or friezes for short) are numerical arrangements satisfying a local arithmetic rule. Conway and Coxeter showed that friezes can be classified by triangulations of polygons. Recently, friezes were actively studied in connection with the theory of cluster algebras, and the notion of a frieze obtained a number of generalizations. In particular, one can define a frieze associated to a bordered marked surface endowed with a decorated hyperbolic metric. I will review the construction and explain how some nice properties can be extended to friezes associated to a pair of pants. The talk is based on a joint work with Ilke Canakci, Anna Felikson and Ana Garcia Elsener.

**Alain VALETTE** (Neuchâtel) :

**Reciprocal hyperbolic elements in  $\mathrm{PSL}_2(\mathbb{Z})$**

An element  $A$  in  $\mathrm{PSL}_2(\mathbb{Z})$  is hyperbolic if  $|\mathrm{Tr}(A)| > 2$ . The maximal virtually abelian subgroup of  $\mathrm{PSL}_2(\mathbb{Z})$  containing  $A$  is either infinite cyclic or infinite dihedral;  $A$  is reciprocal if the second case happens ( $A$  is then conjugate to its inverse). We give a characterization of reciprocal hyperbolic elements in  $\mathrm{PSL}_2(\mathbb{Z})$  in terms of the continued fractions of their fixed points in  $P^1(\mathbb{R})$  (those are quadratic surds). Doing so we revisit results of P. Sarnak (2007) and C.-L. Simon (2022), themselves rooted in classical work by Gauss and Fricke & Klein.