Elliptic genera

EPFL Doctoral Course
(IIIe Cycle Romand de Mathématiques)
15.09.-19.12.2008
Wednesdays 14:15-15:30
see also: http://homeweb1.unifr.ch/dessaia/pub/EllipticGenera/

Abstract: Elliptic genera may be viewed as a generalization of classical operators on finite dimensional manifolds (such as the Signatur and Dirac operator) to the free loop space. In the course we will first review the classical situation (characteristic classes, Hirzebuch formalism for genera, Atiyah-Singer index theorem for signature and Dirac operator, Lefschetz fixed point formula). Then we will introduce elliptic genera focusing on the elliptic genus of level 2 and the Witten genus. We will discuss their main property, the rigidiy under the action of compact connected Lie groups and prove vanishing theorem for these genera. Finally we discuss various applications to Riemannian geometry and manifold topology.

Due to lack of knowledge some very interesting topics will not be discussed, e.g. connections to theoretical physics, connections to elliptic cohomology as initiated by Hopkins-Miller in the late 1980s and the recent work of Stolz-Teichner on elliptic objects and quantum field theory.

Language: Lectures in English, discussion in English/French/German

Objectives: The purpose of the first part of these lectures is to give an introduction to characteristic classes, genera and index theory. In the second part we want to study elliptic genera and their applications to geometry and topology.

Required prior knowledge: basic concepts of topology, basic notions of funtional analysis, undergraduate course in complex analysis (e.g. elliptic functions)

Keywords: vector bundles, characteristic classes, genera, elliptic functions, modular forms, elliptic differential operators, Atiyah-Bott-Segal-Singer index theorems, elliptic genera, rigidity

Literature:

- F. Hirzebruch, T. Berger, R. Jung : *Manifolds and Modular Forms*, Vieweg Verlag (1992)
- Peter S. Landweber et al: *Elliptic Curves and Modular Forms in Algebraic Topology* (Proceedings, Princeton 1986), Lecture Notes in Math. 1326, Springer (1988)
- H. B. Lawson and M.-L. Michelsohn : *Spin Geometry*, Princeton University Press (1989)
- John W. Milnor, James D. Stasheff: *Characteristic classes*, Annals of Mathematics Studies, No. 76. Princeton University Press (1974)
- research articles