A-Fri-Ka Riemannian Topology Research Seminar

Meeting, April 29th 2021 (online)

2:00 - 3:00 pm (CET)

Topology of spaces of metrics generalizing positive scalar curvature (based on joint work with Klaus Kröncke, Hartmut Weiß, Frederik Witt, Olaf Müller and Jonathan Glöckle)

Speaker:
Bernd Ammann (Regensburg)

Abstract:
In this talk I want to summarize some results about some spaces of metrics and their moduli spaces. The metrics we want to study are: Riemanian metrics of positive scalar curvature, Riemannian metrics of nonnegative scalar curvature, structured Ricci-flat metrics, Lorentzian analogues thereof, and Riemannian and Lorentzian manifolds of special holonomy.
A major question of the talk is the passage from strong curvature inequalities such as positive scalar curvature to weak ones such as nonnegative scalar curvature. Recall that if M is a closed spin manifold, then the Atiyah-Singer index theorem and its family version are strong tools to detect non-trivial homotopy groups in the space of positive scalar curvature metrics. These results use the fact that the Dirac operator on a closed spin manifold of positive scalar curvature is invertible. In the case of nonnegative scalar curvature, the Dirac operator is not always invertible, but if it is non-invertible, then the manifold carries a parallel spinor, which implies special holonomy, Ricci-flatness, and virtually abelian fundamental group. In particular, one may detect non-trivial homotopy groups in the space of Riemannian metrics of non-negative scalar curvature, if the fundamental group is not virtually abelian.
We will also discuss spaces of globally hyperbolic Lorentzian metrics on a manifold $M \times (0,1)$, where M is a closed spin manifold. The strict dominant energy condition – which is some Lorentzian or spacetime generalization of positive scalar curvature – leads to an invertible Dirac-Witten operator on any spacelike hypersurface M, and one may use Glöckle's Lorentzian α-index to find non-trivial homotopy groups in the space of such metrics. If we weaken to non-strict dominant energy condition – which is the spactime generalization of nonnegative scalar curvature – generalized Killing spinors take the role of parallel spinors from before. These generalied Killing spinors are related to parallel spinors and special holonomy on Lorentzian manifolds, and imply that the fundamental group is virtually solvable. In particular, we can detect non-trivial homotopy groups in the space of metrics of such Lorentzian metrics, if the fundamental group is not virtually solvable.

3:15 - 4:15 pm (CET)

Geometry of collapsing Ricci-flat Kähler metrics on four manifolds

Speaker:
Ruobing Zhang (Princeton)

Abstract:
This talk focuses on the singularity structure of collapsing of Ricci-flat Kähler metrics. It is known that a generic sequence of Einstein metrics may develop singularities along the convergence. This leads to curvature blowing-up, singular Gromov-Hausdorff limits, and a large variety of non-trivial rescaling limits around singularities (called bubble limits). We will exhibit some discoveries of new collapsing types, more delicate curvature blowing-up behaviors, and more sophisticated bubbling structures. We will also introduce some classification results on the convergence types and the bubble limits.

5:00 - 6:00 pm (CET)

Spin^c manifolds, positive scalar curvature and manifolds with fibered singularities.

Speaker:
Boris Botvinnik (Oregon)

Abstract:
I will discuss a problem of existence of positive scalar curvature on manifolds with fibered singularities. It turns out there are necessary and sufficient conditions for a psc-metric to exist on such objects. There is a particular case of manifolds with fibered singularities when the fiber is a circle. This case leads to psc-metrics spin^c manifolds with special conditions near the singular locus. In particular, I describe some results concerning metrics on spin^c manifolds with positive "twisted scalar curvature", where the twisting comes from the curvature of the spin^c line bundle. This work is joint with Jonathan Rosenberg.