Supervenience and infinitary property-forming operations

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Abstract: This paper provides an account of the closure conditions that apply to sets of subvening and supervening properties, showing that the criterion that determines under which property-forming operations a particular family of properties is closed is applicable both to the finitary and to the infinitary case. In particular, it will be established that, contra Glanzberg, infinitary operations do not give rise to any additional difficulties beyond those that arise in the finitary case.

1 Introduction

Infinitary closure conditions play an important role in debates about supervenience. In particular, they allow us to connect supervenience to other notions, such as reductionism. For instance, by appealing to infinitary Boolean closure Kim established important results for the relation between supervenience and the existence of bridge-laws. He showed that weak supervenience implies that every A-property is coextensive with some B-property, while strong supervenience implies that every A-property is necessarily coextensive with some B-property (cf. Kim: 1993, Ch. 4). Another example is the relation between supervenience and entailment. As McLaughlin and Bennett have shown, "the logical supervenience of property set A on property set B will only guarantee that each A-property is entailed by some B-property if A and B are closed under both infinitary Boolean operations and property-forming operations involving quantification" (McLaughlin and Bennett: 2005, §3.2).

Michael Glanzberg has argued that "the step to infinitary logical operations raises significant metaphysical issues of its own ... [and that] if we accept the use of infinitary logical operations, we still face hard choices about the strength of such operations we should allow" (Glanzberg: 2001, p. 419). In this paper, I will show that his arguments fail and that infinitary property-forming operations do not generate any difficulties in addition to those involved in the finitary case.

2 Supervenience and closure

Supervenience claims specify relations of dependent-variation amongst families of properties. The criterion that determines under which property-forming operations these families of properties are closed is the B-hood-preservation criterion.

> B-HOOD-PRESERVATION CRITERION: accept α-ary operation $\otimes$ iff for any α-tuple of B-properties $F_1 \ldots F_\alpha$, the property formed by applying $\otimes$ to $F_1 \ldots F_\alpha$ is also a B-property.

When assessing whether a particular property-forming operation is to be accepted we need to ask whether this operation can be applied to B-properties to yield further B-properties.[1] As van Cleve notes: "let the base set be closed under a given operation *if and only if B-hood is preserved by that operation*" (van Cleve: 1990, p. 228). For instance, if we are concerned with physical properties, then we should accept property-forming operations that preserve physicality. If the properties that result from applying an operation to physical properties are also physical properties, then the set of physical properties is closed under that operation.

Closure conditions resulting from the B-hood-preservation criterion will have to be decided on a case-by-case basis. Which property-forming operations are to be accepted depends on what B-hood consists in. We have to ask which operations preserve those features that make the properties members of the family of B-properties, whereby these features can be understood as higher-order properties. If the relevant higher-order properties are preserved, then the property set characterised by those higher-order properties is closed under the property-forming operation in question. Put differently, families of properties are characterised in terms of certain higher-order properties, whereby we can have higher-order properties that pick out natural families, such as the higher-oder property of being an intrinsic property, as well as properties that pick out gerrymandered families, such as the higher-order property of having been stipulated to be a member of the B-properties. The question then is which property-forming operations preserve the higher-order properties that characterise B-hood.

Some families of B-properties will be closed under all of the Boolean operations, others only under some of them or even none of them. For instance, the set of intrinsic properties is closed under conjunction, disjunction and complementation, while the set of physical properties fails to be closed under complementation. If the set of B-properties does not form a natural class but simply results from having its members directly specified, then no non-trivial property-forming operations are applicable.

[1] Unless noted otherwise, all claims about base properties (B-properties) apply equally to the supervening properties (A-properties).

3 Infinitary operations

A number of results concerning supervenience, such as Kim's proof that supervenience implies the existence of bridge-laws, can only be established by assuming closure under infinitary operations.[2] Glanzberg has argued that this commitment is problematic and that the infinitary case gives rise to serious new problems and difficulties. I will show that his arguments are mistaken and that no special problems result from countenancing infinitary operations. We can thus agree with Kim when he says: "I don't see any special problem with an infinite procedure here, any more than in the case of forming infinite unions of sets or the addition of infinite series of numbers." (Kim: 1993, p. 152).

In particular, Glanzberg argues that closure under resplicing follows from $\mathcal{L}_{\infty\omega}$-closure (cf. Glanzberg: 2001, p. 424).[3] Resplicing (or diagonal closure) is a property-forming operation proposed by Bacon, which states that "[w]here F_w is the extension of F at world w, and $B_w=\{F_w\colon F \in B\}$, B is also to contain any property G such that $G_w \in B_w$ for each world w" (Bacon: 1986, p. 165). Put differently, if in each world w, G is coextensive with some B-property, then G is also a B-property. Closure under resplicing has been criticised on a number of occasions since it collapses strong supervenience into weak supervenience (cf. van Cleve: 1990, Oddie and Tichý: 1990, and Currie: 1990).

Glanzberg's argument starts by noting that having an infinitary language $\mathcal{L}_{\infty\omega}$ allows us to fully describe a world w by means of the Scott-sentence σ_w.[4] More precisely, σ_w characterises w up to isomorphism if w has countably many elements, and up to partial isomorphism if w has uncountably many elements.[5] Given that in each world w the extension of each B-property can be picked out by some $\mathcal{L}_{\infty\omega}$-formula ϕ_w, we can resplice B-properties by means of the following

[2] Most criticisms of Kim's proof have focused on the issue as to whether it is legitimate to assume closure under complementation. As van Cleve has shown, however, such criticisms are misplaced since Kim's results can be established by means of fewer resources. In particular, closure under infinitary disjunction and conjunction is sufficient since we only need to appeal to B-natures and not to B-maximal properties (cf. van Cleve: 1990).

[3] The infinitary language $\mathcal{L}_{\infty\omega}$ is a first-order language with identity that lacks individual constants and that allows for infinitary conjunction and disjunction, but only allows finite quantifier prefixes.

[4] For detailed and clear accounts as to how Scott-sentences are constructed, cf. Barwise: 1973, pp. 12-24 & Keisler and Knight: 2004, pp. 7-11.

[5] To say that σ_w characterises w up to partial isomorphism is to say that for any world w′ that satisfies σ_w there is a non-empty family $\mathcal{F}$ of mappings such that (i) every mapping $f \in \mathcal{F}$ is an isomorphism that has as its domain a substructure of w and as its range a substructure of w′, (iia) for every mapping $f \in \mathcal{F}$ and every element x of w, there is a mapping $g \in \mathcal{F}$ that extends f and has x in its domain, and (iib) for every mapping $f \in \mathcal{F}$ and every element y of w′, there is a mapping $g \in \mathcal{F}$ that extends f and has y in its range. Karp established that w and w′ are partially isomorphic iff they are $\mathcal{L}_{\infty\omega}$-equivalent (cf. Karp: 1965).

construction:

$$\bigwedge_{w \in W} (\sigma_w \rightarrow \phi_w)$$

This construction lets us conjoin the formulae picking out the extensions of different B-properties in the different worlds, whereby the worlds as well as the extensions of the B-properties are all specified by means of our infinitary language, namely σ_w and ϕ_w respectively. It thereby allows us to take the union of the extensions, in this way forming a respliced property that is in each world coextensive with some B-property. Given that we accept $\mathcal{L}_{\infty\omega}$-closure, the respliced property will also be a B-property.[6]

If resplicing were to follow from infinitary closure, then this would cast doubt on the latter since the former is problematic insofar as it makes weak and strong supervenience equivalent. In particular, resplicing allows us to generate base properties that ensure that objects from different worlds will never be B-indiscernible, thereby collapsing strong into weak supervenience. Accordingly, accepting closure under resplicing ensures that we will not be able to make strong supervenience claims that are capable of capturing interesting determination and dependence relations (cf. Oddie and Tichý: 1990).

While the resplicing operation proposed by Bacon is problematic, infinitary closure is unproblematic since it only implies a restricted version of resplicing which does not collapse the distinction between weak and strong supervenience.

More precisely, in order to show that weak implies strong, we must accept the following lemma (whereby A is free of modal operators and second-order predicates):

$$\Box \exists G_{\in B} A \vdash \exists G_{\in B} \Box A$$

In particular, we need the following instance of this lemma:

$$\Box \forall F_{\in A} \exists G_{\in B} [\forall x(Fx \leftrightarrow Gx)] \vdash \forall F_{\in A} \exists G_{\in B} \Box [\forall x(Fx \leftrightarrow Gx)]$$

This is warranted by resplicing since "if in every world (of some kind) some base property's extension satisfies A, then diagonal closure enables us to splice those extensions together into a new base property with extensions satisfying A in each world (of the relevant kind)" (Bacon: 1986, p. 165).

If weak supervenience holds but strong apparently fails, then there is some x in w and some y in w′, such that x and y are B-indiscernible yet A-discernible. By means of resplicing we can generate a property that ensures that x and y turn out to be B-discernible after all. This is done, for example, by taking the extension in w of some B-property had by x and the extension in w′ of some B-property that y

[6] It should be noted that the issue of contention is not whether these respliced properties exist, but whether they qualify as B-properties. For the purposes of this paper, we will consider properties as sets of possibilia and will accordingly countenance the existence of arbitrarily respliced properties.

lacks. In that way x has a B-property that y lacks, namely the respliced property R, thereby making x and y discernible with respect to B-properties, ensuring that we do not have a counter-model to strong supervenience. Resplicing consequently collapses strong into weak supervenience insofar as the respliced properties ensure that no objects from different worlds will ever be B-indiscernible. This is because there will always be respliced properties with respect to which they will differ. Strong supervenience will consequently trivially hold whenever weak supervenience holds since there will be no B-preserving cross-world mappings.

For this argument to work, we need to accept unrestricted resplicing. The property-forming operation put forward by Glanzberg, however, allows only some degree of resplicing but not unrestricted resplicing. In particular, since the language $\mathcal{L}_{\infty\omega}$ only contains predicates corresponding to B-properties, it follows that the Scott-sentence σ_{w} only describes w up to B-isomorphism.[7,8] As a result, we end up in a situation whereby for any B-indiscernible worlds w and w′, it will be the case that $\sigma_{\mathrm{w}}=\sigma_{\mathrm{w'}}$ and that $\phi_{\mathrm{w}}=\phi_{\mathrm{w'}}$. Accordingly, the B-properties the extensions of which are picked out by ϕ_{w} will be the same as those of $\phi_{\mathrm{w'}}$. Glanzberg's construction cannot pick out the extensions of different properties in B-indiscernible worlds. This implies that B-indiscernible objects taken from B-indiscernible worlds will also be indiscernible with respect to the respliced properties formed by Glanzberg's construction. Yet this means that $\mathcal{L}_{\infty\omega}$-closure does not support our lemma since if w and w′ are B-indiscernible we cannot use ϕ_{w} to pick out the extension of a B-property coextensive in w with a particular A-property, while using $\phi_{\mathrm{w'}}$ to pick out the extension of a different B-property coextensive in w′ with the same A-property. Hence, if it is the case that necessarily for each A-property there is a coextensive B-property, it need not be the case that for each A-property there is a necessarily coextensive B-property. Accordingly, weak supervenience can hold without it being also the case that strong supervenience holds and this means that the restricted resplicing resulting from infinitary closure is unproblematic.

Put differently, the B-indiscernibility of w and w′ ensures that ϕ_{w} and $\phi_{\mathrm{w'}}$

[7] The reason why the predicates of the language are restricted to those that pick out B-properties and why the language does not in most cases contain any names is that the resources of the language need to reflect the features of the property set. That is, there must be a correspondence between the predicates of the language and the properties of the property set that is under investigation, as well as a correspondence between the connectives and quantifiers of the language and the property-forming operations under which the property set is closed. Otherwise, conclusions about closure principles for an infinitary language would not yield any insights regarding infinitary closure conditions for sets of properties. Put differently, the results of an investigation of closure conditions of a language only transfer to closure conditions of property sets if there is a correspondence between the relevant features of the language and of the property set. An important consequence of this is that we cannot introduce names into the language unless the property set contains something that corresponds to them, such as haecceistic properties.

[8] As mentioned above, the Scott-sentence σ_{w} describes w only up to partial isomorphism – in this case up to partial B-isomorphism – if w should have uncountably many elements.

pick out the extensions of the same B-properties in w and w′. This means that ϕ_{w} and $\phi_{w'}$ either pick out extensions in w and w′ of a B-property had by both x and y, or the extensions in w and w′ of a B-property lacked by both x and y. Given the restricted resources of an infinitary language that only has B-predicates, it is not possible to come up with a ϕ-formula that differentiates between w and w′ and allows us to pick out the extension in w of some B-property had by x and the extension in w′ of some B-property that y lacks. Accordingly, x and y will not be discernible by means of the respliced properties which implies that they remain B-indiscernible even if $\mathcal{L}_{\infty\omega}$-closure is accepted and that we thereby have a counter-model to the equivalence of weak and strong supervenience.

Glanzberg also tries to argue that $\mathcal{L}_{\infty\omega}$-closure is problematic since it allows us to generate arbitrary sets and arbitrary properties, thereby not only collapsing strong into weak supervenience, but also threatening to trivialise weak supervenience as well. In particular, if we have "a world where enough combinations of properties allow us to narrow down a single object" (Glanzberg: 2001, p. 425), then we can appeal to the following construction to specify predicates that only hold of exactly one object in that world:

$$\Xi(x) \leftrightarrow \bigwedge \Phi(x)$$

whereby each member of Φ corresponds to a property that is required for narrowing down a single object.

Once we have these predicates the significance of supervenience principles would seem to be threatened. Glanzberg claims that "with the family of Ξ_p we have uniquely individuating properties for all objects, which is a disaster for supervenience" (Glanzberg: 2001, p. 426). If $\mathcal{L}_{\infty\omega}$-closure were to give us uniquely individuating properties for all objects, then supervenience would indeed be in trouble since there would not be any non-trivial B-preserving intra-world mappings and weak supervenience would consequently trivially hold. Yet, this cannot be achieved by means of Ξ-predicates. $\mathcal{L}_{\infty\omega}$-closure only gives us individuating properties for some objects. In particular, we can only uniquely identify an object x in world w by means of B-properties iff there are no objects in w that are B-indiscernible from x. To trivialise weak supervenience, we would have to have individuating properties for all objects in all worlds to which the supervenience claim was meant to apply. This, however, cannot be achieved by appealing to Ξ-predicates constructed in a language that only has predicates corresponding to qualitative properties since such properties only allow us to narrow down a single object in a highly restricted range of cases.[9]

[9]If the set of base properties is allowed to contain non-qualitative properties, such as haecceistic properties, then there is a risk of trivialising weak supervenience. This well-known fact is independent of considerations pertaining to infinitary closure conditions.

Thus, while it is true that individuating properties can be problematic for supervenience claims, problems only arise if the set of base properties contains individuating properties for all objects. The constructions Glanzberg puts forward, however, only allow us to uniquely identify all those objects that do not have B-indiscernible world-mates. In fact, they do not even permit us to do this due to the fact that the vocabulary of the Scott-sentence is restricted to B-predicates.[10] As a result, σ_w does not allow us to uniquely identify worlds, but only allows us to identify equivalence classes of worlds, namely classes consisting of B-indiscernible worlds. This in turn implies that Glanzberg's constructions that invoke σ_w do not allow us to uniquely identify individuals either, but only enable us to identify equivalence classes of individuals, namely classes consisting of B-indiscernible objects taken from B-indiscernible worlds.

Accordingly, we can see that these 'individuating properties' are entirely unproblematic for strong supervenience claims even if the set of B-predicates were sufficiently rich to construct Ξ-predicates for all objects in all worlds. More precisely, weak supervenience would be trivialised if we had Ξ-properties that would allow us to uniquely individuate all objects in all worlds. This is because there would not be any non-trivial B-preserving intra-world mappings, given that every object would always differ from all of its world-mates with respect to Ξ-properties, thereby bringing it about that weak supervenience would trivially hold. Nonetheless, it would not be the case that strong supervenience would be trivialised as well since the Ξ-properties would pick out equivalence classes of B-indiscernible objects taken from B-indiscernible worlds, thereby allowing for B-preserving cross-world mappings that could fail to be A-preserving.

Once we have individuating properties we can construct any arbitrary set P as follows:

$$\bigvee_{p \in P} \Xi_p$$

These arbitrary sets then allow us to add arbitrary properties. We can do this by means of $\mathcal{L}_{\infty\omega}$-formulae ϕ_w that pick out arbitrary sets at each world w and

[10] Glanzberg does not actually invoke Scott-sentences when constructing the Ξ_p-properties. This, however, is necessary since it is only the case that Ξ holds of exactly one object in w, while it may hold of several objects in other worlds. Put differently, these conjunctive Ξ-properties only classify as uniquely individuating properties for certain objects in particular worlds, namely worlds where objects that instantiate such Ξ-properties lack B-indiscernible world-mates, and not as uniquely individuating properties for these objects in all worlds in which they exist, let alone for all objects in all worlds. Accordingly, we need to include a specification of the world in the construction of the individuating properties. In particular, if we want to construct an individuating property for object p in w* then we have to appeal to: $\bigwedge_{w \in W}(\sigma_w \rightarrow \Xi_w)$, whereby this infinitary construction picks out p in w* and the null extension in all other worlds. This way of constructing the individuating properties makes it clear that we are dealing with nothing but a special case of resplicing. (This will be important later on.)

which then feature in the following construction:

$$\bigwedge_{w \in W} (\sigma_w \rightarrow \phi_w)$$

This construction can be considered as an extreme form of resplicing that is equivalent to a property-forming operation that van Cleve calls truncating or trimming and that would lead to absurd consequences for supervenience (cf. van Cleve: 1990, p. 236). Unlike in the case of ordinary resplicing, ϕ_w does not simply take the extension of some B-property, but takes an arbitrary set constructed out of Ξ_p-predicates, thereby seemingly allowing for much more radical resplicing. In particular, if we were able to add arbitrary properties by means of such $\mathcal{L}_{\infty\omega}$-constructions, then we could generate B-properties that would ensure that there would neither be any B-preserving cross-world mappings nor any non-trivial B-preserving intra-world mappings. As a result, both strong and weak supervenience would be trivialised.

However, given the restrictions on specifying arbitrary sets by means of $\mathcal{L}_{\infty\omega}$ that we identified above, the results of adding 'arbitrary' properties turn out to be the same as the results of restricted resplicing. Put differently, given the resources of a language restricted to B-predicates, the sets constructible out of Ξ_p-predicates will be equivalent to the extensions of B-properties over which ϕ_w ranged in the case of resplicing. The properties generated by the restricted resplicing that follows from $\mathcal{L}_{\infty\omega}$-closure are accordingly the same as those that are generated by splicing together extensions of sets that are constructed out of Ξ_p-predicates.

In particular, since Ξ_p-predicates do not pick out individuals but equivalence classes of B-indiscernible individuals that lack B-indiscernible world-mates and which are taken from B-indiscernible worlds, we are neither able to generate any spliced properties that would collapse strong into weak supervenience nor any properties that would trivialise weak supervenience itself. As we saw above, in order for strong supervenience to collapse, we would need to be able to distinguish members of these equivalence classes and include only some into the constructed property. This, however, cannot be done by means of the restricted resources of an infinitary language the vocabulary of which is restricted to B-predicates. Accordingly, $\mathcal{L}_{\infty\omega}$-closure only commits to a restricted form of resplicing that does not generate any problems for supervenience.

It should be noted that even restricted resplicing is problematic for some sets of properties, for instance the set of intrinsic properties. Yet, this is due not to special features pertaining to infinitary operations, but due to Glanzberg's use of Scott-sentences to generate the respliced properties. In particular, appealing to Scott-sentences presupposes that the relevant set of properties be closed under Boolean operations as well as closed under quantification. This presupposition, however, is not warranted in the case of a significant number of families of properties. In other words, there are cases in which Glanzberg's respliced properties are problematic, yet they are problematic not because they are generated by infinitary operations but because they are generated in a way that presupposes property-forming operations that are already problematic in the finitary case. This means that the step to the infinitary case does not raise any further difficulties.

Accordingly, if we are dealing with sets of properties for which we accept Boolean closure as well as closure under quantification, then the restricted resplicing that follows from infinitary closure is unproblematic. When dealing with sets of properties for which we reject Boolean closure or closure under quantification, on the contrary, we cannot appeal to Glanzberg's infinitary constructions to yield restricted resplicing. There is consequently nothing special about the infinitary case. If we accept that B-properties are closed under Boolean property-forming operation $\otimes$, then we should also accept that these properties are closed under infinitary applications of $\otimes$, except in cases in which B-hood is in some sense essentially finitary (which happens either insofar as (i) B-hood precludes the set of B-properties from containing infinitely many members, or insofar as (ii) even though the base set contains infinitely many members, B-properties are essentially such that they apply only to finitely many things (which excludes infinitary disjunctions), or such that they are constructible only via finitary property-forming operations). In the case of these exceptions, we should reject infinitary operations since the B-hood-preservation criterion will tell us that B-properties will not be subject to infinitary closure. This means that we can use the same criterion in the infinitary case as in the finitary case and that, contra Glanzberg, there are no special difficulties or new problems that arise from countenancing infinitary operations.

4 Conclusion

Thus, we have seen that the B-hood-preservation criterion allows us to determine which property-forming operations are to be accepted in both the finitary and the infinitary case. In particular, we saw that infinitary property-forming operations do not give rise to any special problems and that Glanzberg's arguments to the contrary are mistaken. It was established that infinitary closure is unproblematic insofar as it neither collapses strong supervenience into weak supervenience, nor trivialises weak supervenience itself. An infinitary language restricted to B-predicates neither allows us to unrestrictedly resplice properties nor allows us to add arbitrary properties by specifying them in terms of arbitrary sets.[11]

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