

Noumena as grounds of phenomena

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ABSTRACT: The debate between one-world and two-worlds interpretations of transcendental idealism generally proceeds on both systematic and exegetical grounds. A common objection to two-worlds interpretations is that they are committed to an implausible, if not incoherent, ontology and that considerations of charity consequently speak against attributing such a view to Kant. This paper focuses primarily on systematic rather than exegetical considerations. It aims to develop an account of the relation between noumena and phenomena that makes sense of a two-world ontology. In particular, the goal is to elucidate this relation in such a way that it neither undermines noumenal ignorance nor ontologises space and time. Far from being incoherent or implausible, the resulting theory constitutes a powerful metaphysical system.¹

I Introduction

The distinction between noumena and phenomena lies at the core of Kant's transcendental idealism. How exactly this distinction is to be interpreted, however, is much disputed, in particular whether it is a methodological/epistemological distinction or a metaphysical distinction, and, if the latter, whether the contrast is between two types of properties or between two types of objects.² Two-worlds accounts of transcendental idealism operate with a robust metaphysical distinction. They consider noumena to be ontologically distinct from phenomena. Things in themselves and things that appear to us form two disjoint classes of objects. Such a robust distinction takes seriously the idea that transcendental idealism is a form of idealism and provides much-needed resources for accomplishing the tasks that Kant envisages, especially when it comes to making room for transcendental freedom and resolving the antinomies more generally.

Though they are distinct, phenomena and noumena are not unconnected. In particular, they are connected by a grounding relation: noumena are the grounds

¹For a detailed account of the resulting metaphysical system cf. Bader: 2010.

²For a helpful overview of this debate cf. Stang: 2016.

of phenomena (cf. 5:6). Phenomena are derivative entities that owe their existence as well as their determinations to noumena. This is what allows phenomena to inherit their objectivity from noumena and thereby underwrites Kant's empirical realism. Though Kant is committed to a form of idealism, his version of idealism, namely transcendental or formal idealism, is highly distinctive and differs importantly from phenomenalist approaches that reduce objects to mental states. Though phenomena are mind-dependent, they are not dependent on particular minds or their mental states, but are dependent on the forms of intuition that all of us share. Phenomena are matter-form compounds that depend on noumena for their matter, but depend on us for their form (cf. 4:375 & 11:395). Their spatio-temporal form is attributable to the fact that space and time are our forms of intuition. Idealism regarding form is thereby combined with realism concerning matter. It is for this reason that, unlike noumena which are absolutely real and objective in a completely unrestricted sense, phenomena are only real for us (i.e. for beings with our forms of intuition).³ They are empirically real but transcendently ideal.

Not only are the properties of phenomena determined by noumena, the very existence of the phenomenal realm is dependent on noumena. Phenomena are derivative entities. Their matter, which is encapsulated in the manifold of intuition, derives from noumena. This manifold is translated into the spatio-temporal forms of intuition to yield phenomena, i.e. matter-in-form. Since phenomena are constructed out of the translated manifold of intuition, they are not independent existents, but are dependent on noumena. Without noumena there would not be any manifold of intuition, and there would consequently not be anything that could serve as an element in a phenomenal logical complex. Accordingly, not only are the properties of objects within the phenomenal domain supervenient, the domain itself is supervenient.⁴ Noumena determine both the existence and the determinations of phenomena – that they exist and how they exist. Given this dependence of phenomena on noumena, we can infer the existence of noumena from the existence of phenomena. For there to be constructions, there must be something out of which they are constructed. There must be things that can feature as the elements of the logical complexes.⁵

This type of determination and dependence can be precisely characterised by means of supervenience relations. Such relations concern the dependent-variation of properties. They hold if two families of properties are functionally connected.

³They will not appear to cognitive beings with other forms of intuition (who will have their own appearances that are disjoint from ours). Nor will they appear to an intuitive intellect that does not have any forms of intuition – nothing appears to such a non-receptive intellect.

⁴Kim calls this 'existence supervenience': "There is no world in which individuals of D_1 exist but in which individuals of D_2 do not" (Kim: 1993, p. 114).

⁵This inference to noumena is not an inference from effect to cause (which is at issue when inferring the existence of noumena on the basis of our receptivity, which involves positing noumenal affection as the cause of our representations), but from grounded to ground (cf. Bxxvi-xxvii).

More precisely, it is by means of co-ordinated multiple-domain supervenience relations that we can capture in quasi-formal terms the determination and dependence relations between noumena and phenomena.⁶ It will be argued that we are concerned with a multiple-domain, rather than a single-domain, supervenience relation, given that noumena and phenomena are distinct entities that possess different properties and that can be found in different domains (section 2). The fact that noumena and phenomena are connected by the transformation function that is involved in the process of intuition implies that the supervenience relation is mediated and co-ordinated, such that particular noumenal features are connected to particular phenomenal features by means of this transformation function (section 3). Moreover, the base properties are restricted to transcendental properties, since it is only these properties that play a role in yielding phenomena – the others are not compatible with our forms of intuition and are accordingly filtered out (section 4). Finally, it will be shown that this construal of the relation between noumena and phenomena does not violate noumenal ignorance (section 5), and does not involve a problematic ontologising of space and time (section 6).

2 Multiple-domain supervenience

Single-domain supervenience concerns the dependent-variation of different families of properties instantiated by the same objects. It holds if objects that are indiscernible in terms of subvening properties are also indiscernible in terms of supervening properties. This understanding of supervenience is not adequate for capturing the grounding relation between noumena and phenomena in the context of a two-world account, given that phenomenal and noumenal properties have different exemplifiers. Since the subvening and supervening properties are instantiated by distinct things that are to be found in different domains, we have to appeal to multiple-domain supervenience relations. Two distinct and disjoint domains are involved in the supervenience relation, whereby indiscernible distributions of properties in the subvening domain give rise to indiscernible distributions in the supervening domain.

In the case of single-domain supervenience relations, A-properties supervene on B-properties iff B-indiscernibility implies A-indiscernibility across the relevant range of possible worlds. Distributions that are B-indiscernible also need to be A-indiscernible, where distributions are understood in terms of assignments of properties to objects. Two distributions are indiscernible with respect to B-properties if there is a B-preserving isomorphism. A one-to-one mapping Γ from distribution D_1 to D_2 is such a property-preserving isomorphism iff any x in D_1

⁶There is an additional hyperintensional aspect involved in the grounding relation that cannot be captured by supervenience. This hyperintensional aspect can, however, be set aside for the purposes of this paper. In fact, supervenience relations are better suited for our purposes since they allow for greater flexibility and precision.

has any B-property F if and only if the object to which x is mapped by Γ in D_2 also has F.⁷ Thus, A-properties supervene on B-properties iff all B-preserving isomorphisms are A-preserving.

When dealing with a single domain, every B-preserving mapping also has to be A-preserving. This, however, does not make sense when dealing with a plurality of domains. We then have different mappings for the subvening and supervening domains. As a result, B-preserving isomorphisms cannot be A-preserving. This means that we need to specify the relation between the mappings in the different domains if we are to develop supervenience relations across these domains. We need to connect the domains and thereby connect the mappings.

This can readily be done in the case of transcendental idealism. The phenomenal and noumenal domains are co-ordinated and connected rather than being independent of each other. In particular, they are connected by means of the forms of intuition. The forms of intuition give rise to a transformation function that connects the two realms and specifies the way in which phenomenal outputs result from noumenal inputs. This function corresponds to the one involved in the processing of the manifold of intuition. The role of this process is captured nicely by Findlay when he notes that “experience is not a free composition, but rather a translation into the diction of space and time of a text framed in another idiom” (Findlay: 1981, p. 34). The processing functions involved in ordering, processing and synthesising the manifold of intuition in accordance with the forms of intuition can be broken down into three component sub-processes, namely (1) imposition, (2) selection, and (3) translation. Forms of intuition are imposed, the manifold is selected for compatibility with these forms, and the selected manifold is translated into the forms. This ensures that the relation between noumena and phenomena is not direct but mediated by our forms of intuitions. Phenomena are determined by noumena via our forms of intuition.

The forms of intuition thus give rise to a co-ordination relation R that connects members of the subvening domain to members of the supervening domain. The mappings should, accordingly, not be independent of each other, but should track this connection. This can be achieved by appealing to the co-ordination relation in order to specify the images of members of the subvening domain in the supervening domain. This allows us to connect the mappings of the members of one domain with the mappings of their images in the other domain. In this way, we are able to track determination and dependency relations across multiple domains.

Multiple-domain supervenience holds iff every property-preserving mapping on the subvening level is such that the images of the mapped objects under the

⁷If the set of B-properties includes irreducibly plural properties, then the notion of B-indiscernibility must be supplemented by the condition that any plurality of xx's has any plural B-property F iff the image of the plurality under Γ also has F (whereby the image of a plurality is the plurality of the images of the members of the plurality).

co-ordination relation R are also indiscernible. This can be stated precisely by means of the notion of an associated mapping.

ASSOCIATED ISOMORPHISM

A one-to-one mapping of members of the supervening domain Γ' from D_S onto D_{S^*} is an associated mapping of a mapping of members of the subvening domain Γ from D_B onto D_{B^*} , if it is the case that if any collection of members $x_1 \dots x_n$ from D_B is mapped onto $x_1^* \dots x_n^*$ from D_{B^*} by Γ , then Γ' maps the images of $x_1 \dots x_n$ under R in D_S , i.e. $y_1 \dots y_n$, onto the images of $x_1^* \dots x_n^*$ under R in D_{S^*} , i.e. $y_1^* \dots y_n^*$.

This allows us to specify strong global multiple-domain supervenience relations, whereby the associated mappings of all B-preserving isomorphisms must be A-preserving isomorphisms if A-properties are to supervene on B-properties.⁸

SG-MDS for all possible worlds w and w^* , every B-preserving mapping of the members of the subvening domains of w and w^* is such that all its associated mappings of the members of the supervening domains of w and w^* are A-preserving.

Put differently:

A-properties supervene on B-properties iff for all possible worlds w and w^* , every mapping Γ of objects in the subvening domains of w and w^* that is such that any x or plurality of xx 's has any B-property F if and only if the object or plurality to which x or the xx 's are mapped by Γ also has F is also such that any image under R of x or of the xx 's has any A-property G if and only if any image under R of the object or plurality to which x or the xx 's are mapped by Γ also has G .

3 Domain co-ordination

Domains are co-ordinated when there is some relation between the members of the different domains that establishes a correspondence between objects (or plu-

⁸This strong version can be distinguished from a weak version, whereby every B-preserving isomorphism must only have some associated A-preserving isomorphism. Since the weak version fails to track interesting dependence and determination relations we can set it aside.

WG-MDS for all possible worlds w and w^* , every B-preserving mapping of the members of the subvening domains of w and w^* has an associated A-preserving mapping of the members of the supervening domains of w and w^* .

These co-ordinated versions differ only if the co-ordination relation fails to be unique, i.e. if $x_1 \dots x_n$ has a plurality of images under R in the supervening domain. In such cases, a particular mapping of the subvening domain will have a plurality of associated mappings, allowing us to distinguish between a weak version of multiple-domain supervenience that requires only that one of these associated mappings be A-preserving and a strong version that requires all of them to be A-preserving.

ralities of objects) in the supervening domain and objects (or pluralities of objects) in the subvening domain. This co-ordination relation connects the mappings in the different domains that are used for assessing for property-preserving isomorphisms when dealing with multiple-domain supervenience relations. It thereby allows us to make sense of determination and dependence relations between the noumenal and phenomenal realms.

The co-ordination relation is particularly crucial when dealing with non-holistic forms of determination. If the determination is local, such that particular noumena (or pluralities thereof) determine particular phenomena (or pluralities thereof), then there must be some way of connecting them up and specifying which phenomena correspond to which noumena. However, it is also required if determination should be holistic, as long as it matters not only which properties are instantiated but also which objects are doing the instantiating. Otherwise, all that one can say is that if the subvening domain of noumena is a certain way, then the supervening domain of phenomena is a certain way (where each property distribution is characterised completely independently of the other). This is because one can then only say that there exists an A-preserving mapping of the members of the supervening domain if there exists a B-preserving mapping of the members of the subvening domain. This, however, means that the base is unable to distinguish between any of the permutations of the supervening domain and hence cannot fix which object has which properties, but can only fix that there are some objects instantiating the properties in question.

Incorporating a co-ordination relation into our supervenience principle allows us to connect the domains in a non-holistic manner. By connecting particular members of the different domains, we can model the fact that particular phenomena are grounded in particular noumena.⁹ This is important since the phenomenal sphere is likely not to be determined holistically by the noumenal sphere, but instead result from a more localised form of determination that connects particular objects in the two domains. Even though a holistic determination of the phenomenal sphere as a whole is not ruled out in principle, as far as theoretical reason is concerned, more fine-grained determination relations are necessary when bringing in commitments stemming from the practical side, given that they are required to make sense of the idea that a subject's empirical character is a reflection of its intelligible character (cf. A₅₄₀/B₅₆₈).

The role of the co-ordination relation is particularly clear when considering the multiple-domain supervenience version of individual as opposed to global supervenience:

Phenomenal properties supervene on noumenal properties relative to co-

⁹As we will see, such a co-ordination relation is not too restrictive and does not threaten to undermine noumenal ignorance, since the mere condition that it connects members of the domains does not determine whether a one-one, one-many, many-one, many-many or variably polyadic connection holds.

ordination relation R just in case for any collections of phenomenal objects $x_1 \dots x_n$ and $y_1 \dots y_n$ that have images under R and any possible worlds w and w^* , if $R|x_1 \dots x_n$ in w is indiscernible with respect to noumenal properties from $R|y_1 \dots y_n$ in w^* , then $x_1 \dots x_n$ in w is indiscernible with respect to phenomenal properties from $y_1 \dots y_n$ in w^* .

The supervenience relation that characterises the relation between the noumenal and phenomenal realms is a co-ordinated supervenience relation, where the domains are connected by means of the translation function of the process of intuition. The forms of intuition thus feature in the supervenience relation insofar as they provide the translation function that gives rise to the co-ordination relation. This function translates the matter provided by noumena into phenomenal properties, into a spatio-temporally ordered manifold, thereby giving rise to the phenomenal ‘images’ of the noumenal entities.¹⁰ The fact that we impose certain frameworks, namely space and time, into which the information contained in the manifold of intuition is translated ensures that noumena and phenomena are co-ordinated and that the supervenience relation is mediated. The particular translation function associated with these frameworks then determines the precise nature of this mediation, i.e. the way in which the co-ordination takes place.

The co-ordination relation, moreover, enables us to individuate the relata of the supervenience relation. On the face of it, our ignorance of noumena would seem to preclude us from giving an adequate account of how noumenal property distributions are individuated. Yet, providing such an account is necessary for making a supervenience claim. This is because supervenience holds if B-indiscernibility implies A-indiscernibility, i.e. if the fact that the base properties are distributed in the same way implies that the supervening properties are also distributed in the same way. Since assessing for indiscernibility requires us to appeal to property-preserving isomorphisms, we need to specify noumenal mappings. Yet, it is not clear what the members of the domain are that should be mapped. These could be noumenal objects, certain aspects of such noumenal objects, or some other individuating characteristic such as noumenal analogues of spatio-temporal locations. There seems to be no way for us to provide an exhaustive list of the possible candidates, nor any principled way to decide between them. We simply do not know how to individuate noumenal grounds.

This problem can be addressed by appealing to the co-ordination relation. Rather than providing a direct characterisation of the way in which noumenal property distributions are individuated, which seems to be a futile endeavour given noumenal ignorance, we can characterise the individuation indirectly by

¹⁰When there is a one-many co-ordination relation that connects one phenomenal feature to a plurality of noumenal features, there will not be a unique noumenal feature that is the image of x and that can be mapped. Instead, the image of x will be a collection of features and one has to assess the collection for indiscernibility by mapping its members. The same holds, *mutatis mutandis*, for many-one and many-many relations.

means of the co-ordination relation. We can do this by providing an account of the individuation of phenomenal property distributions and then letting the co-ordination relation pick out the noumenal analogues of these individuating features. An indirect account specifies how to individuate phenomenal properties and then appeals to the images under the co-ordination relation of these individuating features in order to identify the relevant unit of analysis.¹¹ Whether the noumenal entities or items that are selected in this way turn out to be objects or aspects of objects or something altogether different can be completely left open.¹² Hence, all we need to do is to give an adequate account of the individuation of distributions of phenomenal properties. The co-ordination relation will then take care of specifying in an indirect manner the corresponding noumenal individuating features.

How then are phenomenal property distributions to be individuated, and what are the members of the distributions which are mapped when assessing for property-preserving isomorphisms? Two candidates suggest themselves, namely (i) spatio-temporal regions and (ii) phenomenal objects. When the phenomenal realm is considered on its own, it is ontologically amorphous. All there is is a spatio-temporal distribution of intensive magnitudes that can be fully characterised by means of the categories of quantity and quality. All the ontological structure of the phenomenal realm is derived from the noumenal realm. There are, accordingly, no bounded and unified individual objects at the phenomenal level when it is considered in abstraction from its noumenal grounds. We should thus not use a metaphysically substantive notion of objecthood that involves the category of substance in characterising property distributions.

Instead, we should either use a non-substantive notion of objecthood or appeal to spatio-temporal distributions. These two options turn out to be equivalent since, according to the minimal understanding of objects, phenomenal objects can be understood as filled spatio-temporal regions.¹³ An object in this minimal sense is a collection of properties local to a spatio-temporal region. Properties are distributed across regions and objects are then identified with these filled regions.¹⁴ An individuation in terms of the minimal notion of objecthood, whereby

¹¹There is no need for all of the entities instantiating the properties in the base domain to be of the same kind, i.e. the images under R can be heterogeneous.

¹²Since we are unable to give a transparent specification of R, we only know that there is a correct individuation, without being able to transparently specify what it is. As a result, noumenal ignorance is not undermined.

¹³These minimal objects are contrasted with ontologically substantive phenomenal objects, i.e. with phenomenal substances, which require the properties that are bundled into a minimal object to be bounded and unified as a result of being adequately grounded.

¹⁴This minimal understanding of an object has close analogues in field theories as well as in supersubstantialistic theories. Transcendental idealism allows us to reap the benefits of this minimal notion of an object without incurring the costs that this notion brings with it in the context of these other theories. For example, while field theories as well as supersubstantialistic theories have the problematic consequence of making all minimal objects modally rigid, we can account

a phenomenal object is considered as a filled spatio-temporal region, thus coincides with an individuation in terms of spatio-temporal structures of property instantiations. Supervenience, accordingly, holds if we have indiscernible filled spatio-temporal regions, given that the images of those regions are indiscernible.

Finally, the co-ordination relation plays a crucial role by mediating the grounding relation between noumena and phenomena. Even though facts as to what the world is like for us are determined by facts about noumena together with facts about our forms of intuition, the forms of intuition do not enter into the supervenience base. Instead of entering into the base, the translation function deriving from our forms of intuition mediates the supervenience relation. This means that we do not reduce phenomena to the conjunction of noumena and the forms of intuition, but rather reduce phenomena to noumena via the forms of intuition. Conversely, we generate phenomena out of noumena via forms of intuition, rather than out of the conjunction of noumena and forms of intuition. Matter and form are not on a par but play different roles. The grounds of phenomena are restricted to their matter, whilst their form do not play a grounding-role. The forms of intuition are not amongst the grounds but mediate the grounds to result in hylomorphic matter-form compounds. In particular, instead of being ingredients in the reductive base, they mediate the relation between noumena and phenomena by specifying the selection, filtering and translation processes that are applied to the manifold of intuition.

As a result, the existence of phenomena does not presuppose that there are any subjects with our forms of intuition. All that is required for the existence of phenomena is the existence of the relevant kinds of noumena. We need nothing more than noumena to get phenomena. In other words, noumenal properties exhaust the supervenience base. Since the forms of intuition do not enter into the supervenience base, the supervenience claim does not need to be restricted in such a way as to hold only in cases where the forms of intuition are present. This is what ensures that transcendental idealism involves a distinctive form of mind-dependence, whereby phenomena are not dependent on any particular minds or their mental states, but are instead dependent on the forms of intuition. The role that our noumenal selves play is not to function as grounds of the phenomenal realm (except when it comes to grounding the corresponding phenomenal selves), but to fix which transformation functions are applicable. The contribution that we make by supplying the forms of intuition is thus not a contribution at the level of the grounds.

for the modal flexibility of minimal objects. This can be achieved without undermining the modal rigidity of spatio-temporal regions and without bringing in any dubious resources such as a counterpart-theoretic account of modality. This is possible because transcendental idealism provides us with two ways of individuating objects, namely (i) in terms of their phenomenal features, and (ii) in terms of their noumenal grounds. Only the former way coincides with the rigid way of individuating spatio-temporal regions. The latter version is independent of spatio-temporal characteristics, thereby providing us with the requisite flexibility.

4 Transcendent and transcendental properties

So far, we have indiscriminately referred to the noumenal sphere as constituting the supervenience base of the phenomenal sphere. By appealing to the process of intuition, we can give a more fine-grained account of the base properties as well as of the objects belonging to the subvening domain. This process allows us to distinguish between various kinds of noumenal properties and restrict the set of subvening properties to a proper sub-set of the noumenal properties. In particular, it is the selection function of the process of intuition that is responsible for demarcating those properties which play a role in grounding phenomena and making experience possible and which we can label the transcendental properties from those properties that play no such role and which we can label the transcendent properties.

More precisely, the selection function determines which properties count as transcendent properties. Whether a property is transcendent depends on whether the instantiation of that property can function as a ground of noumenal affection whereby a manifold of intuition is provided that is suitable for being processed by cognitive beings having our forms of intuition. Properties classify as transcendent either if they provide a manifold that is incompatible with the forms of intuition and that is consequently filtered out, or if they are inert and do not give rise to a manifold at all. Such transcendent properties do not enter as input into the translation function and are in that respect irrelevant to the genesis of the phenomenal sphere. They are consequently also irrelevant to the supervenience relation that models the grounding relation between noumena and phenomena. Since they do not affect phenomena, insofar as phenomena do not immediately depend on them and are not determined by them, they do not belong into the supervenience base. We can accordingly restrict the supervenience base such that phenomenal properties supervene only on a sub-set of the properties of the noumenal realm, namely on transcendental rather than transcendent properties.

There may well be relations amongst objects or properties within the noumenal realm that would make some, or maybe even all, transcendental properties dependent on certain transcendent properties. This would, for instance, be the case if everything were to ontologically depend on God. That is, for all we know, it might be the case that there are internal or external necessary connections in the noumenal realm. These connections would ensure that any nomologically or logically possible world (depending on what kind of necessity is involved in the necessary connections) that contains the transcendental properties would also contain those transcendent properties that are connected to these transcendental properties by means of these connections.

This, however, does not affect the supervenience principle, which states that indiscernibility in terms of transcendental properties implies indiscernibility in terms of phenomenal properties. It will still be the case that if the same tran-

scendental properties are instantiated, then the same phenomenal properties will also be instantiated. All it implies is that it might be nomologically or logically impossible to have a possible world that contains an isolated duplicate of the transcendental realm. In other words, it might not be possible to have a duplicate of the transcendental realm without there being transcendent properties of certain kinds (if we have generic dependencies) or without there being particular transcendent properties (if we have rigid dependencies).

In order to get a supervenience principle that takes the restriction of the supervenience base into account, we have to assess for indiscernibility with respect to transcendental properties. The images under R , namely $R|x_1 \dots x_n$ and $R|y_1 \dots y_n$, have to be indiscernible only with respect to transcendental properties in order for indiscernible phenomenal properties to arise, rather than having to be indiscernible with respect to noumenal properties in general. Indiscernibility in terms of transcendental properties suffices for making objects B-indiscernible, given that transcendent properties are excluded from the supervenience base. Accordingly, as long as $R|x_1 \dots x_n$ and $R|y_1 \dots y_n$ are indiscernible in terms of transcendental properties, x and y must be indiscernible with respect to phenomenal properties, even if $R|x_1 \dots x_n$ and $R|y_1 \dots y_n$ differ in transcendent properties.

All the subvening properties are members of the set of transcendental properties. As a result, the co-ordination relation R connects phenomenal objects and properties to a sub-set of the noumenal realm, namely the transcendental realm. This means that any $R|x_1 \dots x_n$ is an object with transcendental properties. Objects that only have transcendent properties do not feature in the subvening domain, on the basis that they do not (immediately) ground phenomena.

Phenomenal properties supervene on transcendental properties relative to co-ordination relation R just in case for any collections of phenomenal objects $x_1 \dots x_n$ and $y_1 \dots y_n$ that have images under R and any possible worlds w and w^* , if $R|x_1 \dots x_n$ in w is indiscernible with respect to transcendental properties from $R|y_1 \dots y_n$ in w^* , then $x_1 \dots x_n$ in w is indiscernible with respect to phenomenal properties from $y_1 \dots y_n$ in w^* .

Alternatively:

Phenomenal properties supervene on transcendental properties relative to co-ordination relation R just in case for any possible worlds w and w^* , every mapping Γ of transcendental objects from w and w^* that preserves transcendental properties is such that all its associated mappings of phenomenal objects from w and w^* preserve phenomenal properties.

These supervenience claims reflect the core features of the grounding relation between noumena and phenomena, insofar as they are irreflexive, strong cross-world

supervenience claims, that hold with logical necessity for all possible worlds,¹⁵ connecting properties across multiple domains, whereby the domains are co-ordinated and whereby we assess the indiscernibility of supervenience bases in terms of transcendental properties.

5 Noumenal ignorance

Since the phenomenal and noumenal realms are connected to each other and stand in co-ordinated supervenience relations, it may be wondered why we cannot gain knowledge of noumena on the basis of our knowledge of phenomena. Given that phenomena and noumena correspond to each other and given that there is a metaphysical connection between these two realms, one might think that there is also an epistemological connection that would allow us to identify the noumenal counterparts of phenomena, thereby undermining noumenal ignorance.

To begin with, it is worth noting that our knowledge of phenomena is rather limited. In particular, knowledge of non-structural features is ruled out. We only know phenomenal relations, whilst the underlying non-relational properties, namely the physical intensive magnitudes, are not accessible to us. We can only specify them indirectly in terms of the effects that they can bring about (cf. R5590). However, we cannot make an inference from these effects to the underlying categorical properties that ground the relevant causal powers. This is because phenomenal roles might be multiply realisable and because the possible realisers can only be specified in an opaque but not a transparent manner. Though we do know that there must be a role filler, our knowledge is restricted to knowing that the role is filled by something or other. Accordingly, we never know what it is that fills the role. Multiple things could fill this role and there is no way for us to single out the actual role filler. In a sense, our ignorance is ineffable, to borrow Lewis's phrase, since we do not even know what the different options are (cf. Lewis: 2009, pp. 215-216). That is, we do not even know what the different possible realisers are. Even if there were only one possible candidate for filling the role, we would not know what would be filling the role and would not be able to characterise this unique realiser. This is because we are not acquainted with the realiser and lack the conceptual resources to give a transparent specification of the realiser. Instead, we can only specify it opaquely as whatever it is that fills the role in question.¹⁶

Moreover, even if we were able to transparently specify the phenomenal role

¹⁵Since phenomena are not emergent entities that require bridge laws to come into existence, but are logical complexes that have as their elements translated matter provided by noumena, the relation between noumena and phenomena is one of logical supervenience, such that B-indiscernibility implies A-indiscernibility in all possible worlds rather than only in a restricted range of worlds where certain bridge laws obtain.

¹⁶The distinction between transparent and opaque specifications is due to Foster: 1982, p. 62.

filler, this would not threaten noumenal ignorance since we cannot appeal to the inverse of the translation function. This is because we lack knowledge of the way that the translation function works and are not able to reverse engineer the process whereby phenomena arise. Since we do not know the input-output relationships, we can only characterise the inputs as whatever it is that gives rise to the phenomena in question. Yet, we cannot specify them in a transparent manner. In addition, the problem of multiple realisability arises again. Supervenience need not be symmetric: though phenomena supervene on noumena, noumena need not supervene on phenomena. Phenomena can be indiscernible even though their images under R are discernible. This means that we cannot get from phenomenal outputs to the noumenal inputs that generate them, but only to the range of possible inputs. Accordingly, we cannot construct the base domain on the basis of the supervening facts. Instead, all we can do is use R to pick out the noumenal image of some phenomenal outputs. In that case, one does not try to reverse-engineer the inputs from the outputs, but takes the base domain as being given and picks out the actual ground in this domain. The problem of multiple realisability is then circumvented, since, even though a given output can have multiple possible grounds, it has a unique actual ground and this is the image under R .

Knowledge of the phenomenal realm is thus restricted to structural knowledge. The problem now is that a sufficiently strong connection between noumena and phenomena might underwrite an inference from knowledge of the structure of the phenomenal realm to knowledge of the structure of the noumenal realm. According to Russell, noumenal ignorance is compromised since noumena can be understood as forming “a world having the same structure as the phenomenal world, and allowing us to infer from phenomena the truth of all propositions that can be stated in abstract terms and are known to be true of phenomena” (Russell: 1920, p. 61; also cf. Schlick: 1979, p. 104).

Given the transcendental ideality of space and time, it follows that there is no spatio-temporal correspondence between the realms. The non-spatio-temporality of noumena prevents us from getting from the spatio-temporal structure of phenomena to that of noumena. As a result, the noumenal structure that could be inferred from phenomenal structure would only be abstract mathematical structure, rather than concrete spatio-temporal structure. The structure of the phenomenal world, accordingly, could at most allow us to identify the cardinality of the noumenal domain. Unless concrete relations can be identified and suitably connected to known phenomenal relations, any collection of the right cardinality will have the relevant abstract structure, making this kind of knowledge almost trivial.¹⁷

¹⁷This problem for pure versions of structuralism, known as the Newman problem, was first identified by Newman: 1928 and Carnap: 1928, §§153-155. It can be overcome by transcendental idealists, since the forms of intuition allow one to operate not with abstract mathematical

Yet, even knowledge of the cardinality of the noumenal domain is ruled out. There is no guarantee that the noumenal and phenomenal realms have the same cardinalities. Indeed, given the possibility of transcendent properties and objects any isomorphism claim is almost certain to be false. Transcendent objects do not play any role in grounding experience and thereby ensure that the noumenal realm has additional structure that is not reflected in phenomena. Because of this surplus structure, the structure of phenomena should not be expected to be isomorphic to the structure of noumenal reality, undermining any cardinality claim.

It might be suggested that instead of an isomorphism claim about the relation between phenomena and noumena, we can make an embeddability claim by specifying a sub-set of the noumenal realm, namely the realm consisting of transcendental objects, that stands in an isomorphism relation to the phenomenal realm. We could thereby make cardinality claims about transcendental objects, while lacking knowledge of the noumenal sphere within which the transcendental objects are embedded. This suggestion, however, is also problematic since we do not know how to individuate the members of the domains and since we are ignorant of the translation function that connects the transcendental realm to the phenomenal realm. There are a number of plausible principles of individuation even when it comes to individuating the members of the phenomenal domain. We could appeal to filled spatio-temporal regions, logical complexes, or bounded phenomenal objects. With respect to noumena, things are even less clear. The members of the domain could be analogues of regions, they could be noumenal objects, aspects of such objects or something altogether different. As we have seen, noumenal ignorance implies that the individuation of noumena is best characterised indirectly via the process of intuition, i.e. via the co-ordination relation. Since this relation could be a many-one, one-many, many-many or variably polyadic relation, there is no guarantee that we can identify any isomorphisms and any cardinality claim turns out to be unfounded.

Even though there is a non-arbitrary connection between the phenomenal and noumenal realms, and even though phenomena somehow correspond to and are manifestations of noumena, any isomorphism or embeddability claim is inappropriate. Instead of making such claims, we should argue that the two domains are co-ordinated, without specifying the precise character of the co-ordination relation. We only specify this relation functionally insofar as it is equivalent to the translation function that results from our forms of intuition. However, we do not specify the particular input-output relationships. This allows for sufficient flexibility to allow not just for one-one determination or correspondence as would be required for an isomorphism, but also for one-many, many-one, many-many and variably polyadic relations.

structure but concrete spatio-temporal structure, thereby vindicating structural realism within the phenomenal realm, cf. Bader: 2010, chapter 2.2.3.

This method only guarantees isomorphisms if we take the units of individuation, which are then mapped by the isomorphisms, as being equivalent to the input and output variables of the translation function. There would then be one output for each input, guaranteeing a one-one relation. This, however, does not say anything about whether these inputs and outputs are simple or complex or whether they can be given more fine-grained individuations. In particular, the individuation principles for the different domains might well turn out to be completely different. This would give rise to disparate units for the two domains, thereby undermining the significance of the isomorphism claim. There is thus no reason to assume that the correct individuation principles imply a one-one function that gives rise to an isomorphism between phenomena and transcendental objects. We are, consequently, ignorant of the cardinality of the transcendental realm as well. By contrast, if one were to adopt a one-world view, then noumenal ignorance could not be preserved. A commitment to an identity claim between noumena and phenomena would imply a one-to-one correspondence ensuring that the cardinality of transcendental objects could be determined on the basis of the cardinality of the phenomenal realm, which would at the same time also determine a lower bound on the cardinality of the noumenal realm.

Thus, even though phenomena are grounded in noumena, the fact that we are dealing with a co-ordinated supervenience relation that is based on certain translation and filtering processes has the consequence that there is no perfect mirroring relation between the realms and that we are not able to make determinate inferences about the latter on the basis of the former.

6 Ontologising space and time

Phenomena are derivative entities that are grounded in noumena. They are consequently reducible to noumena via the forms of intuition. To this it might be objected that there are irreducible relations at the phenomenal level, that certain phenomenal relations are non-supervenient and that this implies that we cannot consider all aspects of the phenomenal realm as being supervenient.

If some phenomenal relations should be irreducible, then this would indeed undermine the claim that the phenomenal sphere supervenes on the noumenal sphere. Yet, although there is a sense in which certain phenomenal relations are indeed irreducible, this turns out to be unproblematic since they are reducible in another sense. More precisely, there are phenomenal relations, such as spatio-temporal relations, which cannot be reduced to the intrinsic properties of their relata. These relations occupy a fundamental position within the ontological inventory of the phenomenal realm. This, however, is unproblematic, given that the fact that something is not reducible to phenomenal properties does not imply that it is not reducible tout court. Though these relations cannot be reduced to any other phenomenal items, they are nonetheless reducible to noumenal fea-

tures. While they do not supervene on phenomenal properties, they do supervene on noumenal features.¹⁸

There is nothing that precludes noumenal relations from featuring in the supervenience base of the phenomenal realm. While phenomenal relations might not supervene on noumenal non-relational properties alone, it seems reasonable to claim that they do supervene on a base that includes noumenal relations. Thus, if the ‘non-supervenient’ phenomenal relations are to be grounded, it is not sufficient to ground the intrinsic properties of their relata. Instead, we need to provide a direct ground of these relations. Accordingly, we should claim that phenomenal relations that do not supervene on the intrinsic properties of their relata supervene on noumenal relations. This means that the supervenience principle must include relations in the supervenience base. Unless we want to be extreme Leibnizians and argue that all relational properties are reducible to non-relational properties, we will have to make room for noumenal relations upon which the ‘non-supervenient’ phenomenal relations can supervene.

Relational and non-relational phenomenal properties thus supervene on relational and non-relational noumenal properties. We must take the relational properties of the noumenal grounds into consideration if we are to have a supervenience principle that covers all phenomenal features. Accordingly, the phenomenal sphere is wholly sustained by noumena. It is a logical complex and every aspect of it, including every relational aspect, is reducible to that out of which it is constructed.

The reducibility of phenomenal relations, including the reducibility of spatio-temporal relations, may seem objectionable on the basis that such a reduction of phenomenal relations could be considered to amount to ontologising space and time in the way done by Leibnizians and that Kantians should be suspicious of such a commitment. To ontologise space and time amounts to treating them as transcendently real, making them properties of things in themselves. Understood in this way, we can see why Leibnizians ontologise space and time. Even though they argue that monads are atemporal and aspatial, they nonetheless accept the reducibility of spatio-temporal relations. If spatio-temporal relations are reducible to monads, then they are features of monads, even if monads themselves are neither in space nor in time. They are not fundamental features of monads, but are rather derivative features. This means that the reducibility of spatio-temporal relations implies that space and time turn out to be features of monads, that is, of things in themselves.

The Kantian can avoid treating space and time as transcendently real, while accepting the reducibility of relational properties. This combination of views is possible because of the imposition and translation functions involved in the

¹⁸How we ought to understand the reducibility of chiral relations and what impact this has on the argument from incongruent counterparts will not be discussed here. An investigation of this intriguing topic will have to wait for another occasion.

process of intuition. These functions mediate the supervenience relation. They consequently also mediate the reducibility relation. In both the Leibnizian and the Kantian system there is supervenience and reducibility. Whereas the relation between monads and phenomena in the Leibnizian system is characterised by a non-mediated reducibility relation, in that spatio-temporal relations are immediately reducible to determinations of monads, the supervenience relation in the Kantian system is a co-ordinated multiple domain supervenience relation, whereby the co-ordination relation is provided by the process of intuition. This ensures that, even though the Kantian system also includes a commitment to a supervenience relation, the supervenience relation in this system is mediated by the translation function. Rather than understanding phenomena as just being confused perceptions of noumena, they are logical complexes that result from a transformation process that translates the information contained in the manifold of intuition into imposed frameworks, namely the forms of intuition. Accordingly, in order to reduce spatio-temporal relations we need both (i) the supervenience base and (ii) the transformation functions. This means that we are dealing with a mediated reducibility relation.

The difference between the two systems accordingly derives from the forms of intuition. These forms ensure that we have real heterogeneity, rather than merely confused perceptions. The forms of intuition genuinely add something and thereby ensure that we do not end up with direct reducibility to the properties of noumena. We need something in addition to the non-relational properties of noumena to get phenomena. We even need something in addition to the non-relational and relational properties of noumena to get phenomena. What we need is the translation function. It is the translation function that provides the connection between noumena and phenomena, thereby mediating the supervenience and reducibility relations. The forms of intuition ensure, in this way, that the mediated reducibility of spatio-temporal relations does not involve the ontologising of space and time.

From a Kantian point of view, the kind of reducibility of relational properties to which Leibnizians are committed is particularly problematic, given that Leibnizians hold a relationalist view of space and time. In the context of such a relationalist view, the reduction of spatio-temporal relations amounts to a reduction of space and time themselves. This is something the Kantian cannot accept. Space and time would thereby become relational properties of things in themselves. For the Kantian, however, space and time are forms of intuition. They are not reducible to noumenal relations. Instead, they are our contributions to the phenomenal realm. It is precisely for this reason that the structures of space and time are not contingent but necessary and can be known by us a priori.¹⁹

¹⁹The structure of space and time is built into the supervenience relation, since the co-ordination relation is based on the forms of intuition that mediate the relation between noumena and phenomena. As a result, the structural properties of space and time trivially supervene – they

Whilst the Kantian needs to reject the reducibility of space and time, there is no need to reject the reducibility of spatio-temporal relations. We can reduce spatial relations without reducing space itself. All that needs to be rejected is the relationalist view of space and time. Instead, the Kantian can claim that space and time are mental frameworks. As such, they are independent of any spatio-temporal relations. Things are related within space and time, rather than space and time merely being the systems of spatio-temporal relations.

According to transcendental idealism, spatio-temporal relations are grounded in and supervene on noumenal relations. As a result, we can reduce spatio-temporal relations to noumenal relations. This reduction, however, has to go via the forms of intuition, thereby avoiding worries of ontologising space and time. To understand how this works, we need to appeal to the co-ordinated supervenience relation and to the role of the translation function. This allows us to retain our commitment to the view that space and time are nothing but forms of intuition, whilst accepting the reducibility of all phenomenal relations.

7 Conclusion

Phenomena are distinct from noumena, but are nevertheless closely connected to them, in that they are grounded in noumena via the forms of intuition. This grounding relation can be modelled by means of a co-ordinated multiple-domain supervenience relation, whereby the co-ordination relation plays a crucial role in preserving noumenal ignorance and avoiding a commitment to ontologising space and time.²⁰

supervene on any base, even an empty base, and in that sense are necessary.

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