# Kant on logic*

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ABSTRACT: Immanuel Kant developed a highly systematic and nuanced conception of logic that differs significantly from, and is in important respects superior to, the logics developed by his predecessors and contemporaries, as well as those in use nowadays. Whilst his logic plays a crucial role in both his theoretical and practical philosophy, it is unfortunately widely misunderstood, leading to deep-rooted misinterpretations of the Critical philosophy. This chapter provides an account of the logical forms of judgement in general and transcendental logic, spelling out the different ways in which concepts can be synthesised to form categorical judgements and the ways in which categorical judgements can be synthesised to form hypothetical and disjunctive judgements.

## 1 Introduction

Logic is the science of correct thinking (cf. Bviii-ix). The ultimate building blocks of our thoughts are concepts. These are general representations that represent objects mediately. Unlike singular representations, which represent their objects immediately, concepts represent mediately by means of conceptual marks that represent features that can be shared by different objects. The content of a concept consists of the marks that are contained in that concept. The extension of a concept, by contrast, consists in that which falls under that concept. This can be understood either in terms of the subordinate concepts that fall under it (= logical extension), or in terms of the objects instantiating it (= non-logical extension). What falls under a concept is that from which the concept can be abstracted, namely the more determinate concepts or things that differ in various ways, whereby one abstracts from these specific differences when forming the

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concept in question (cf. JL 9:96). The concept's content determines its extension. The extension of a concept is the intersection of the extensions of the conceptual marks contained in the concept. Extension and content are thus inversely related: the more a concept contains, the more specific it is, so that less falls under it (cf. JL 9:95).[1]

Concepts can be synthesised to make judgements. Judgements relate different concepts to each other. The different ways in which concepts can be combined to form categorical judgements, and the ways in which categorical judgements can be combined to form complex judgements, are the forms of thought that are enumerated in the table of the logical functions of judgement (cf. A70/B95). These forms represent the fundamental types of synthesis that can be applied to generate judgements. The resulting judgements can then be related to each other to make inferences, which includes both immediate inferences of the understanding and mediate inferences of reason. This chapter provides an account of the logical forms of judgement, spelling out the different ways in which concepts and judgements can be synthesised in general and transcendental logic.[2]

## 2 General logic

General logic is entirely formal. It abstracts from all content and considers only formal features of and structural relationships amongst concepts and judgements. Concepts, which are the constituents from which the domain of general logic is formed, are treated as basic unanalysable givens. Since concepts differ in terms of their content, yet general logic abstracts from content, all concepts are considered to be interchangeable. Judgements that involve different concepts but have the same form are treated in exactly the same way by general logic.

In pure general logic there are eight forms of judgement. There are two forms under each of the four headings, with each dichotomy established by means of an analytic division that corresponds to the concepts of reflection (cf. A260-268/B316-324).[3]

[1] Given that there is no conceptum infimum (cf. JL 9:97), every concept has an infinite extension. The idea that less falls under a concept that contains more than another is hence to be understood in terms of the extension of the former being a subextension of the latter. (This applies straightforwardly to concepts that are higher/lower than one another, but runs into difficulties in case of concepts on different branches of the concept hierarchy (cf. JL 9:98).)

[2] The fourth heading 'modality' (which is not part of the content of judgements but concerns acts of judging) will be set aside. For a detailed treatment of logical modality, cf. "Kant's theory of modality" Bader: manuscript, chapter 1.

[3] For a helpful discussion of the relation between the forms of judgement and the concepts of reflection cf. Longuenesse: 1998, pp. 131-163.

| | | |
|---|---|---|
| | **Quantity** | |
| | universal | |
| **Quality** | particular | **Relation** |
| affirmative | | categorical |
| negative | **Modality** | hypothetical |
| | problematic | |
| | assertoric | |

## 2.1 Categorical judgements

Categorical judgements are simple judgements that form the basis of complex judgements, namely hypothetical judgements (as well as disjunctive judgements in the context of transcendental logic). They involve two concepts that are combined by means of concept subordination, whereby one concept is subsumed under the other. The concept that is subsumed is the subject of the judgement and the concept under which it is subsumed is the predicate of the judgement. The headings 'quantity' and 'quality' characterise categorical judgements.

'Quantity' concerns the extension of the subject concept. Either the entire extension of the subject concept is subsumed under the predicate concept, or an indeterminate part thereof is subsumed. In the former case, the predicate applies without exception to the subject, in which case the judgement is a universal judgement. In the latter, the judgement is a particular judgement that admits of exceptions.

'Quality' concerns the valence of the copula. A categorical judgement synthesises the constituent concepts and asserts a subordination between them. The synthesis of these concepts is represented by the copula. This synthesis takes two valences, namely inclusion and exclusion that are represented by 'is'/'is not'.[4] When the subject is included in the extension of the predicate, the two concepts are positively combined, resulting in an affirmative judgement. When the subject is excluded from the extension of the predicate, the two concepts are negatively combined, resulting in a negative judgement.

Quantity and quality together give rise to the four possible forms of categorical judgements: 1. universal affirmative, 2. universal negative, 3. particular affirmative and 4. particular negative.[5] The basic unit of judgement is thus: 'all/some S is/is not P'.

Categorical judgements take the subject to ground the predication (cf. JL 9:121). The predication is taken to correspond to the marks that inhere in the subject. In the context of general logic, the marks that inhere in the subject

[4] Universal negative judgements are best represented by 'all S are not P' rather than 'no S is P' since they involve the exclusion from P (represented by 'are not P') of the entire extension of S (represented by 'all S').

[5] Each of them can be judged either problematically or assertorically (and in transcendental logic also apodictically).

are the marks that are predicated of it. Whereas grounds of inclusion underlie affirmative judgements, grounds of exclusion underlie negative judgements. The subject grounds an affirmative judgement when the predicate that is affirmed of the subject picks out one of these mark. In case the predicate that is denied of the subject is excluded by one of the marks inhering in the subject, the subject grounds a negative judgement.[6]

These grounding connections underlie categorical syllogisms. In an inference one asserts the conclusion on the basis of the subsumption of the condition of this judgement under a general rule. A categorical syllogism subsumes the subject, via the middle term, under the general rule, namely the grounding connection asserted in the major premise. The grounding connection then transfers via the middle term to the conclusion, allowing the subject to be a ground of the predication.

For instance, the major premise 'all humans are mortal' establishes a grounding connection between the *terminus medius* <human> and the *terminus major,* <mortal>. The minor premise 'Socrates is human' connects the *terminus minor* <Socrates> to the *terminus medius*, thereby transfering the grounding connection asserted in the major premise to <Socrates>, such that <Socrates> is a ground of <mortal>, which underwrites the conclusion 'Socrates is mortal'. Put differently, by subsuming <Socrates> via the minor premise under <human>, which according to the major premise is a ground of <mortal>, <Socrates> becomes a ground of <mortal>. The major premise establishes that <mortal> is a mark of <human> and the minor that <human> is a mark of <Socrates>, allowing for the inference to 'Socrates is human' in accordance with the principle of affirmative categorical syllogisms that the mark of a mark is a mark of the thing of which it is a mark (*nota notae est nota rei ipsius*).[7]

In transcendental logic, which is not entirely formal but takes into consideration the content of concepts, what inheres in the subject is not restricted to what is predicated of the subject but also includes what is contained in the subject. If P is contained in S, then S is a logical ground of P such that the judgement 'S is P' is logically true, i.e. it is an analytic truth. The content-sensitivity of transcendental logic thereby makes room for containment relations amongst concepts and hence for the distinction between analytic and synthetic judgements. Analytic judgements are then no longer restricted to those in which the predicate is identical to the subject but also include all those in which the predicate is identical to or excluded by a part of the subject.

The possibility of categorical judgements rests on the law of non-contradiction. This law is not to be understood in terms of the negation of the conjunction con-

[6]Negative judgements can also be vacuously true. Their vacuous truth lacks a ground and is not based on exclusion but based on the subject term being empty such that it is trivially excluded from the extension of the predicate.

[7]The principle of negative categorical syllogisms is *repugnans notae, repugnat rei ipsi*.

sisting of a judgement and its negation, nor in terms of the impossibility of a judgement being both true and false. Rather, it is a principle of logical possibility, which states that one cannot predicate something that contradicts the subject: "No thing has a predicate that contradicts it" (A151/B190). It is a constitutive condition of judgements, according to which the attempted combination of a subject concept with a predicate fails to constitute a judgement when they are combined in a way that generates a contradiction (e.g. the attempt to judge that S is not S).[8] Subject and predicate cancel each other out when they are combined in a contradictory manner, such that one does not succeed in forming a judgement. In short, it does not concern the negation of a conjunctive claim, but the impossibility of a predication that contradicts the subject.[9]

The truth of categorical judgements can take two forms. Kant distinguishes between the logical truth of judgements, which concerns the formal relationships amongst their constituents, and the material truth of judgements, which concerns correspondence to the world.[10] Logical truth concerns logical extensions: the logical extension of S has to be suitably related to the logical extension of P. An affirmative (/negative) judgement is true if the relevant part of the logical extension of the subject is included in (/excluded from) the extension of the predicate. Material truth, by contrast, concerns non-logical extensions. A categorical judgement is true if the objects and properties falling under the concepts are related to each other in the way that the judgement represents them as being related. The property picked out by the predicate concept has to be instantiated by (or, in the case of negative judgements, fail to be instantiated by) the relevant part of the non-logical extension of the subject concept.[11] This means that the

[8] 'S is not S' involves an overt contradiction. In transcendental logic, which considers the content of concepts, there are also covert contradictions. These involve predications that contradict a part of the subject: e.g. 'S is not P' where P is contained in S, as well as predications that contradict a subject, not by excluding it from an extension to which it belongs, but by including it in the complement of that extension: e.g. 'S is non-S'.

[9] Negation in negative judgements applies to the copula and hence cannot have a conjunction of judgements within its scope. The logical modality 'impossible', by contrast, applies to judgements as a whole.

[10] Since they are world-involving, material truths are synthetic truths. Since material truth is not accessible to logic, logic cannot determine whether synthetic judgements are true, but can only determine what follows from their being true, i.e. how these judgements enter into inferences. (Analytic/synthetic is here applied to truths and falsehoods, rather than to judgements. Even the material truth of an affirmative analytic judgement is synthetic, given that it has existential commitments. The truth of such a judgement requires S to have a non-empty non-logical extension. This means that things that are S have to exist, which is a synthetic matter. Though negative analytic judgement do not have existential commitments, since they can be vacuously materially true when there is no S, that there is no S is a synthetic matter as well, as long as S is not a contradictory subject term. Accordingly, only when the emptiness of S is established on analytic grounds is the vacuous material truth of negative judgements an analytic matter.)

[11] Accordingly, in the context of material truth, the inherence of marks amounts to the instantiation of properties.

non-logical extensions of the two concepts have to overlap in the right way. A universal affirmative judgement, for instance, is materially true if S has a non-empty non-logical extension and all its members instantiate P, not as a matter of logic, but as a matter of fact.

The ontological commitments of categorical judgements are determined by the quality of the judgement and the type of truth that is at issue. Only the material truth of affirmative judgements (including infinite judgements) implies existential commitments. This holds independently of the quantity of the judgement. No matter whether the affirmative judgement is universal, particular or singular, the subject must have a non-empty non-logical extension in order to be materially true.[12] Things that are S have to exist for them to be included in the extension of P (or its complement non-P). When there is no S, then both 'S is P' and 'S is non-P' are materially false. One cannot affirm anything (not even something negative, namely non-P) of S when the non-logical extension of S is empty. An empty extension cannot be subsumed. By contrast, the material truth of negative judgements 'S is not P' (and 'S is not non-P') implies no existential commitments. Such judgements are vacuously materially true when they involve empty terms. It is then trivially the case that the (non-existing) members of an empty non-logical extension are excluded from the extension of both P and of non-P.

The fact that affirmative judgements are materially false when S has an empty non-logical extension implies that logical and material truth come apart. They come apart, not only in the straightforward sense that there can be material truths that are not logical truths, but also in the more radical sense that a judgement can be logically true yet materially false. This implies that logical truth is not sufficient for material truth. This is because the logical extension of S can be included within P, without its non-logical extension being included in P, when its non-logical extension is empty and thus cannot be subsumed. For instance, the judgement 'all unicorns have horns' is analytically true, yet materially false.

This can also happen in the case of non-empty non-logical extensions. The particular judgement 'some S is P' can be logically true, yet materially false, even though S is non-empty, as long as all existing S are not P. No part of the non-logical extension of S is then included P, yet a part of the logical extension of S is included in P. Although Kant sometimes suggests that only universal judgements can be analytic (cf. "All analytic judgements are universal" R3083, 16:649), this is incorrect.

First, the universal judgement 'all S are P' entails the particular judgement 'some S is P'. Subalternation is a valid immediate inference of the understanding (cf. JL 9:116). Similarly, conversio per accidens is a valid immediate inference (cf. JL 9:118), so that 'all S are P' entails the particular judgement 'some P is S'.

[12] Particular judgements are not existentially loaded and are not to be interpreted in terms of existentially quantified judgements (cf. Bader: 2021).

In both cases the particular judgement will be analytic if the universal judgement from which it follows is analytic.

Second, if S is logically compatible with both P and non-P, then neither 'all S are P' nor 'all S are non-P' are logical truths, so that neither of these universal judgements is analytic. S can then be logically divided in terms of these opposed predicates. The concept generated by adding P as a differentia, namely SP, contains P as a mark, such that 'all SP are P' is an analytic universal judgement. Since the analytic judgement 'all SP are P' entails the particular judgement 'some S is P', the latter is likewise analytic, since what logically follows from an analytic judgement is itself analytic.

Third, even though P is neither contained in S nor excluded by S when S is compatible with both P and non-P, facts about containment and exclusion, in particular facts about non-containment and non-exclusion, ground the logical truth of the particular judgements 'some S is P' and 'some S is not P'. The logical compatibility of S and P as well as of S and non-P follows from the fact that P is neither contained in nor excluded by S. Though one does not find P by analysing S, one does not find non-P either. If a predicate P is neither contained in nor excluded by a subject S, then the logical truth of 'some S is P' and of 'some S is not P' follows analytically, such that both of these particular judgements are analytic despite their corresponding universal judgements not being analytic.

In the same way that the material truth of affirmative judgements presupposes that the subject term has a non-empty non-logical extension, since there would otherwise not be any objects to which the concept applies, the logical truth of affirmative judgements presupposes that the subject term has a non-empty logical extension. When the subject term is contradictory, there is, strictly speaking, no subject concept. This means that nothing can contain this 'concept'. The 'concept' will not even contain itself.[13] The logical extension is then empty, so that both affirmative and negative assertions, i.e. both 'S is P' and 'S is non-P', are false, in accordance with the rule *non entis nulla sunt praedicata* (A792-793/B820-821). The logical truth of an affirmative judgement presupposes that the subject is not a *non entis*, i.e. a contradictory concept. S must have a non-empty logical extension for this extension to be subsumable under P (and likewise for non-P) such that the affirmative judgement that predicates P of S can be logically true. Negative judgements, by contrast, are vacuously true when S is contradictory.

The claim that a judgement with a contradictory subject term can be vacuously true might seem to conflict with Kant's claim that conformity with the law of non-contradiction is a constitutive condition of categorical judgements. A 'judgement' with a contradictory subject term would seemingly fail to satisfy

[13] If there were to be lowest concepts, something that Kant denies, then the logical extension of such concepts would be empty when construed in terms of proper containment but would nevertheless be non-empty due to the concept improperly containing itself, which would underwrite the analytic judgement 'S is S'.

this constitutive condition. This would make it impossible for there to be negative judgements that could be vacuously true, such that the truth of negative judgements would likewise presuppose their logical possibility.

The law of non-contradiction, however, requires that the predication does not contradict the subject. That the subject term itself is contradictory does not imply a contradiction between subject and predicate. The predication need not contradict the subject and the constitutive condition of categorical judgements can hence be satisfied. There is an important difference between a contradictory judgement, such as 'S is not S', which, strictly speaking, is not a judgement at all, and a judgement involving a contradictory subject term.[14] When the predication contradicts the subject, it is not possible to combine the component concepts into a judgement. No synthesis can take place and hence no judgement can be formed. By contrast, when a collection of incompatible marks cannot be combined into a concept, the intersection of the extensions of these marks will be empty and this empty extension will trivially not be part of the extension of the predicate, rendering negative judgements involving such contradictory subject terms vacuously true. Accordingly, although the modal contrast between truth and falsity is a division of possibility, such that it precludes impossibility, truth and falsity only presuppose the logical possibility of the judgement, not, however, of the subject term.

## 2.2 Hypothetical judgements

A judgement synthesises its components and thereby asserts a conditioning relationship amongst them, insofar as one of them is subordinated to the other.[15] In the case of categorical judgements, this relation is internal to the judgement. Such judgements synthesise two concepts with the subject being the condition of the predicate. In the case of hypothetical judgements, it is an external relation relating different judgements. A hypothetical judgement is a complex judgement that involves an external conditioning relation, in particular a consequence relation that is represented by the connection 'if …then …', which relates different judgements to each other. The antecedent is judged to be a ground of the consequent, such that the truth of the former grounds the truth of the latter, which makes it possible for hypothetical judgements to function as major premises of hypothetical syllogisms.

Complex judgements are based on other judgements and are ultimately made

[14] Since 'S is not S' violates a constitutive condition of judgements, it is not truth-apt and hence is not vacuously true when S is contradictory. (Similarly, 'S is S' violates the law of non-contradiction when S is contradictory and so will not be truth-apt – accordingly, it will not even be false. For this reason, the *non entis* rule does not commit Kant to a form of dialetheism, contra Proops: 2021, p. 253.)

[15] The components of a disjunctive judgement are coordinated due to being mutually subordinated.

up out of categorical judgements (cf. Prol. 4:325 fn.).[16] Quantity and quality do not pertain to complex judgements but only to the categorical judgements out of which they are composed. The contrast between universal and particular judgements only applies to the component judgements – hypothetical judgements themselves do not have a quantity. Similarly, the consequence relation itself does not have a valence and cannot be negated. What can be negated is the predication in the antecedent or the consequent.

Hypothetical judgements composed of two categorical judgements have the form: 'If 'all/some S is/is not P', then 'all/some Q is/is not R". Their component judgements are determined with respect to all four headings in the table of the logical functions of judgement. Otherwise, they would not be able to play their intended role in hypothetical syllogisms. For it to be possible to assert the antecedent as the minor premise of a hypothetical syllogism so that the consequent can be inferred as the conclusion, it has to be possible to detach the component judgements, so that they can function as free-standing judgements. This means that the antecedent and consequent of the hypothetical judgement constituting the major premise have to correspond in their logical form to the minor premise/conclusion (and only differ in terms of their modality). Since such syllogisms have quantified assertoric judgements as their minor premise and quantified apodictic judgements as their conclusion, the component judgements likewise have to be quantified judgements.[17]

A hypothetical judgement takes the antecedent to be a ground of the consequent. If the antecedent is a logical ground of the consequent, then the consequence is logically true and the hypothetical judgement is an analytic judgement.[18] By contrast, if it is a real ground, then it is materially true and the hypothetical judgement is a synthetic truth. In that case, the consequence obtains, such that the antecedent grounds the consequent, not as a matter of logic, but as a matter of fact. What is required for the truth of the judgement is that the component judgements are connected in the right way, where this can either be a formal or a material consequence (cf. R3264, 16:747), which is possible even if both the antecedent and the consequent are false (cf. JL 9:105-106).

[16] Kant's discussion of dilemmas (cf. JL 9:130) shows that complex judgements can be made up out of other complex judgements, e.g. a hypothetical judgement can have a disjunctive judgement as its consequent.

[17] One often finds statements like: 'If S is P, then Q is R' in the literature. These are incomplete due to ignoring the quantity of the component judgements.

[18] It is often claimed that Kant's account of analytic judgements, understood in terms of concept containment, is restricted to categorical judgements. This claim is misguided – hypothetical and disjunctive judgements can equally be classified as analytic or synthetic (cf. "Kant's theory of modality" Bader: manuscript, chapter 1).

# 3 Transcendental logic

In transcendental logic, there are four additional judgement forms. This is because transcendental logic is not entirely formal but takes into consideration the content of our concepts.[19] Under each heading a third judgement form is added, with each trichotomy constituting a synthetic division into 1. condition, 2. conditioned, and 3. that which arises out of the combination of condition and conditioned (cf. B110; 10:366-367). These additional forms are not distinguished in general logic but are treated as being equivalent to one of the prior two forms. In the case of quantity and quality, the third form is indistinguishable from the first (i.e. singular judgements are treated like universal judgements and infinite judgements like affirmative judgements). In the case of relation and modality the third is indistinguishable from the second (i.e. disjunctive judgements are treated like hypothetical judgements and apodictic judgements like assertoric judgements).

| | **Quantity** | |
|---|---|---|
| | universal | |
| | particular | |
| **Quality** | singular | **Relation** |
| affirmative | | categorical |
| negative | | hypothetical |
| infinite | **Modality** | disjunctive |
| | problematic | |
| | assertoric | |
| | apodictic | |

## 3.1 Singular judgements

Transcendental logic countenances singular judgements as a distinct judgement form. General logic, by contrast, treats singular judgements in the same way as universal judgements since they enter into syllogisms in the same way, given that singular judgements, like universal judgements and unlike particular judgements, do not admit of exceptions (cf. A71/B96). This makes it possible for categorical syllogisms composed exclusively of singular judgements to be valid, without violating the *ex puris particularibus nihil sequitur* rule, which requires at least the major premise, which asserts a rule, to be universal (cf. JL 9:124).

Importantly, singular and universal judgements play the same role only in the two types of syllogisms that general logic recognises, namely categorical and hypothetical syllogisms. They play a different inferential role in transcendental

[19] Sometimes the transition to transcendental logic is said to take place when one leaves behind the logical functions of judgement and turns to the categories. This, however, is the transition from the logical employment to the real employment of our faculties. Transcendental logic is already to be found in the table of the logical functions of judgement.

logic, where disjunctive syllogisms are recognised in addition. This is because disjunctive syllogisms are based on the opposition of the component judgements and singular judgements differ from universal judgements in terms of their opposites (cf. section 3.3). The singular judgements 'the S is P' and 'the S is non-P' are opposites such that the disjunctive judgement 'the S is P or the S is non-P' involves an exhaustive and exclusive division of S and can hence function as the major premise of a disjunctive syllogism. By contrast, the universal judgement 'all S are P' is not opposed by the universal judgement 'all S are non-P', since these are contraries that can both be false, such that one cannot infer the truth of the one from the falsity of the other. Instead, it is opposed by the particular judgement 'some S is non-P'.

Kant's predecessors operated with a distinction between singular concepts and abstract concepts (cf. Meier: 1752, §301). Singular judgements involve a singular subject concept, whereas general judgements involve an abstract subject concept, whose extension can be subsumed in whole (= universal judgement) or in part (= particular judgement). Kant, however, rejects the existence of singular concepts. For him, all concepts are general by their very nature. The difference between singular and universal judgements thus does not lie in the nature of the concepts that constitute these judgements. Instead, it lies in the way in which these concepts are used. Whilst rejecting the existence of singular concepts, Kant does recognise the singular use of a concept (cf. Thompson: 1972). A singular judgement is one involving the singular use of the subject concept. What exactly this amounts to, however, is unclear.

What a singular judgement involves at the level of non-logical extensions is clear. A singular judgement is a judgement about a single thing, i.e. one object is judged to fall under the predicate. What it involves at the level of logical extensions, however, is far from clear. This question is usually not even raised, since such judgements are standardly taken to be essentially about objects (relatedly they are often taken to be essentially bound up with intuition and thus to move beyond the purely conceptual). This approach, however, is misguided. The contrast between logical and non-logical extensions applies equally to singular judgements as it does to universal and particular judgements. They can be evaluated with respect to both logical and material truth. The singular judgement 'the S is S', for instance, is analytic and logically true, even if S does not actually exist such that the judgement is materially false. Moreover, universal judgements entail singular judgements (cf. "Under the universal, the particular and singular are all contained" R3171, 16:693), such that universal judgements that are logically true imply the logical truth of the singular judgements that they encompass.[20] Accordingly, we need an account of singular judgements at the level of logical

[20] Relatedly, singular judgements have the same their existential commitments as universal and particular ones. In each case, it is only the material truth of affirmative judgements that has existential commitments.

extensions.

This, however, is difficult. Given Kant's commitment to there not being lowest concepts (cf. JL 9:97), every concept has a logical extension encompassing subordinate concepts. Given the transitivity of 'falling under', i.e. anything that falls under S′ also falls under S if S′ is under S, it is not possible to simply pick out some member of its logical extension. Transitivity ensures that any claim about a member S′ of the extension of S will also be a claim about the members of the logical extensions belonging to the extension of S′ and hence will not be a singular claim.

This difficulty can be overcome by construing the singular use of a concept as treating the concept as if it were a singular concept. Kant repeatedly states that the subject concept of a singular judgement does not have an extension and can be considered as a point (cf. A71/B96; JL 9:102; R3068, 16:639-640; DWL 24:755). Since there are no singular concepts but only singular uses, this can only mean that the concept is treated as if it were a conceptum infimum (cf. JL 9:97). One treats the concept as not having a logical extension, i.e. not having any subordinate concepts. One considers the concept not as a whole that is composed of parts, i.e. a whole with subordinate concepts as its parts, but merely as a part, i.e. as something that is judged to be a part of P without itself being a whole, such that one does not subsume under the predicate everything that falls under the subject concept (i.e. the whole including all its parts) but simply the concept itself as a point (such that it is merely a part, cf. JL 9:102).

This means that what one does in terms of subsumption is the same when making universal and singular judgements – in each case the subsumption does not admit of exceptions. The difference between these judgements lies in the subject concept, though not in what this concept does in fact contain (since the very same concept is used), but in the way in which it is used and the content that is treated as having. The singular use of a concept treats it as if it were completely determined and as not being part of the content of any other concept.

Some concepts are, as a matter of convention, only used in a singular way (cf. JL 9:97). This is how Kant construes proper names. They are linguistic devices that we use to indicate that, on the basis of a convention, a concept is exclusively given a singular use. In order for this to work in practice, it has to be the case that, as a contingent matter of fact, there are not too many, ideally only one, existing members of S (in the vicinity of those who are party to the convention).[21] This means that the level of determination involved in these concepts has to be sufficiently complete for the context to suitably restrict the extension.

Other concepts are sometimes used in a singular way and sometimes in a general way. When it is used singularly, one treats the concept as if it were a singular concept in just the same way as one does when using a concept that is

[21] This relative/local uniqueness is not part of the mechanism that makes judgements involving this concept singular, but a precondition for the possibility of a successful convention arising.

exclusively used in a singular way.[22] Such a singular use is represented by 'the S', which indicates the supposed uniqueness of S, insofar as one takes S to have no extension and to pick out only one object.[23]

Singular use appears to be continuous with particular and universal use at the level of non-logical extensions. In each case one operates with the extension of the subject concept and picks out different parts thereof, by making a claim about the whole extension (= universal), an indeterminate part thereof (= particular), or a particular member thereof (= singular). When considering logical extensions, however, an important difference becomes apparent. Universal and particular judgements treat the concept as the general concept that it is and operate with its extension. They use the concept in abstracto, i.e. as an abstract/general representation. By contrast, singular judgements use the concept in individuo, i.e. as a concrete/individual representation, treating it as if it were a singular concept.

There are two ways of construing non-logical singularity. In each case, the judgement is about one object: in one case the object is treated as a point in an extension and in the other case as a point-sized extension. The former treats universal, particular and singular as being continuous with each other: in each case the judgement is operating with the same extension, either picking out the whole extension, or a part thereof, or a point in that extension. The latter, by contrast, treats universal and particular judgements, which operate with the extension of S, differently from singular judgements, which operate with a truncation of S that reduces the extension of S to a point. The member of the non-logical extension of S is then not considered as one amongst many but as constituting the extension of S. Rather than picking out one object in the extension of S, one treats S as if that object were its entire extension.

This latter approach is required to underwrite Kant's claims about syllogisms. General logic treats singular and universal judgements interchangeably. Every syllogism involving universal judgements that is valid in general logic has to remain valid when substituting corresponding singular judgements (cf. A70/B96). This means that the validity of the syllogism Barbara:

| | |
|---|---|
| 1. | all P are Q |
| 2. | all S are P |
| ∴ | all S are Q |

implies the validity of its singular counterpart:

[22] This means that the convention story is simply a special case of singular use, whereby the concept is always given a singular use. Accordingly, there are not two forms of singular use (contra Rosefeldt: 2000, p. 110).

[23] Rosefeldt: 2000, p. 115 fn. 180 notes that Kant rarely uses deictic formulations and mainly operates with the definite article. This is rather puzzling on the restricted quantification reading, but can be explained by the singular concept reading in terms of the supposed uniqueness that singular use implies. On this view, deictic expressions only work indirectly by indicating which object the singular use of a concept picks out, but are not part of what makes for a singular use.

1. the P is Q
2. the S is P

∴ the S is Q

This syllogism, however, is not valid on what we can call the restricted quantification reading, according to which a singular use of a concept picks out one member of its extension. On that interpretation, the major premise would say of a member of P that it is in Q and of a member of S that it is in P, without any guarantee that the P in the major premise is identical to the S in the minor premise, yet that is required to validate the conclusion that the S is Q. Put differently, there is no guarantee that the two points in the extension of P coincide, since the point corresponding to the S that is in P and the point that corresponds to the P that is in Q could be different.

The singular concept interpretation proposed here, by contrast, underwrites the validity of the syllogism since the two premises together ensure that both S and P are given a point-sized interpretation. If both concepts themselves are points, then the predication in the minor premise implies that S and P coincide such that both fall in the extension of Q, given the predication in the major premise, so that the conclusion does follow.

Accordingly, what is at issue is not the number of things being subsumed by a judgement, but whether the judgement holds without exception, given that syllogistic inferences involving singular judgements are not allowed to admit of exceptions.[24] Both the interpretation in terms of a point within the extension of S and the interpretation in terms of S itself being point-sized agree that the judgement is about one object, but only the latter rules out exceptions. If S itself is not (treated as) singular, then a claim about a single member of S admits of exceptions, i.e. is compatible with other members of S being subsumed under different predicates. By contrast, the absence of exceptions is guaranteed when S itself is (treated as) singular, i.e. when the concept's extension itself shrinks to a point. The concept then falls in its entirety either inside the extension of the predicate or outside it, as also happens in the case of universal judgements. "A singular concept has no sphere, it is a point and and thus has to fall either wholly outside the sphere of the predicates or wholly inside it. Consequently, singular judgements are equivalent to universal judgements." (24:463)

It is often thought that singular judgements are bound up with intuitions (e.g. Longuenesse: 1998, p. 139). The most radical version of this view understands the singularity of singular judgement in terms of the singularity of intuitions, construing the subject of a singular judgement, not as a concept, but as an intuition (e.g. Kiesewetter: 1798, p. 64). The subject of a singular judgement, however, cannot be an intuition. This would imply that the predicate could likewise be an

[24] This is also supported by the alignment of the forms of quantity with the concepts of reflection.

intuition, given that "Anything can function as a *logical predicate*; even the subject can be predicated of itself" (A598/B626), yet Kant is clear that concepts are predicates.[25] Singular judgements like all categorical judgements, are composed of two concepts. Transcendental logic, just like general logic, operates only with concepts, not with intuitions. Intuitions only come in when moving from the logical employment of our faculties to their real employment, not when moving from general logic to transcendental logic.

Rather than considering intuitions as components of singular judgements, the singularity of intuition is often construed to be that which makes singular use possible by allowing us to pick out individual objects. Whilst it can certainly play such a role, intuition is not essential for singular judgements. One can make singular judgements about things that are not and even cannot be given in intuition, such as objects in the distant past, non-existing objects, fictional objects and supersensibles. Since universal judgements entail the singular judgements that they encompass, a universal claim such as 'all noumena are non-spatio-temporal' entails singular judgements about noumena, despite the fact that such singular judgements involve objects that cannot be given in intuition. Whilst establishing the objective validity of such judgements concerning supersensibles poses serious difficulties, they are, from the point of view of logic, entirely unproblematic and classify as singular judgements just as much as ones concerning objects given in intuition.

## 3.2 Infinite judgements

Infinite judgements involve a form of negation that differs from that involved in negative judgements. Infinitising negation concerns not the copula but the predicate. Instead of excluding the subject from the extension of the predicate, subsumption under the negative predicate non-P amounts to including the subject in the extension of the complement of P. They are called 'infinite' since including something in the complement of P amounts to including it in any of a possibly infinite number of predicates that are incompatible with P.

From the perspective of general logic, infinite judgements are indistinguishable from affirmative judgements (cf. A71-72/B97). In the same way that singular judgements play the same role in syllogisms as universal judgements, infinite judgements play the same role as affirmative judgements.[26] Since infinite judgements involve inclusion, they can function as minor premises of valid categorical syllogisms, without violating the *ex puris negativis nihil sequitur* rule, according to which nothing follows from premises that are wholly negative and which requires

[25] Also cf. Korte & Repo: 2011, p. 392 and Land: 2013, p. 225 on the importance of having a concept in subject position for traditional syllogistics.

[26] Differences between infinite and affirmative judgements arise in the context of transcendental logic when it comes to disjunctive syllogisms, since they can be used to form contradictorily opposed judgements.

the subsumption in the minor premise to be affirmative, such that something can be subsumed under the rule that is asserted in the major premise (cf. JL 9:124).[27]

The relation between subject concept and predicate concept is the same in infinite and affirmative judgements. Both of them include the subject in the predicate and hence are to be construed in terms of the positive copula 'is'. For this reason both are false when S is empty (they are materially false when the non-logical extension is empty and logically false when the logical extension is empty), since an empty extension cannot be included in the extension of any predicate. The difference between these judgements concerns not the connection between subject and predicate but the nature of the predicate itself, namely whether it involves a positive mark P or a negative mark non-P. This difference between positive and negative predication is only accessible in transcendental logic since it concerns the content of the predicate concept. "Logic does not look at the content, but the form (relation) with respect to quality. Thus *negationes* of the predicate do not yield negative judgements" (R3035, 16:626).

Whilst being alike to affirmative judgements in terms of the valence of the copula, infinite judgements are alike to negative judgements in that S is excluded from P. Yet, whereas a negative judgement does nothing more than exclude S from P, an infinite judgement also includes S in the complement of P. This difference ensures that, even though 'S is non-P' entails 'S is not P', since including S in the complement of P also involves excluding it from P, the converse does not hold. 'S is not P' does not entail 'S is non-P' except when S is non-empty. When S is empty, the infinite judgement 'S is non-P' is false, whereas the negative judgement 'S is not P' is vacuously true. This is also why the judgement 'S is not non-P', which excludes S from the complement of P, is a negative judgement rather than an infinite judgement, despite involving the negative mark non-P.[28] This judgement entails in transcendental logic the affirmative judgement 'S is P', yet it does so only when S is non-empty (since S then has an extension that is excluded from the complement of P and is thus in the extension of P). When S is empty, by contrast, 'S is not non-P' is vacuously true.

Infinitising negation might seem to be subject to a form of double negation elimination insofar as 'S is non-Q' where Q = non-P is equivalent to 'S is P'. This is because including S in the complement of the complement of P is the very same thing as including S in P. Though the complement of the complement of P is indeed P, infinitising negation is not an operator that can be iterated. If an infinite judgement were simply one that involved including S in the extension of a predicate to which a 'non-' operator is applied, then the judgement 'S is non-Q' would be an infinite judgement, yet would be equivalent to the affirmative

[27] For a helpful account of the *ex puris negativis nihil sequitur* rule in traditional syllogistics and Kant's reflections on this rule cf. Jesiołkiewicz: 2020, chapter 5.

[28] The fact that 'S is not non-P' is not an infinite but a negative judgement ensures that infinite judgements and negative judgements are mutually exclusive (pace Stang: 2012, p. 1122 fn. 12).

judgement 'S is P', which would undermine the claim that these are fundamentally different forms of judgement. If 'non-' were a negation operator that could be applied to any predicate, there would be no genuine difference between affirmative and infinite judgements. These judgements would merely differ in terms of how predicates are picked out. Judgements would then be affirmative or infinite only in a relative but not an absolute way. Where P and Q are complements of each other, Q is negative relative to P, i.e. Q = non-P, yet P is negative relative to Q, i.e. P = non-Q. The inclusion of the very same subject extension in the very same predicate extension, would once come out as an infinite judgement and once as an affirmative judgement.[29]

For Kant there is a fundamental difference between positive and negative marks. The latter have an infinite extension. In fact, strictly speaking, a negative mark does not have an extension at all. As Kant notes: "that it belongs in the sphere outside A, which is not really a sphere at all" (JL 9:104). A negative mark indirectly picks out an infinite disjunction of marks whose members are picked out negatively in terms of being incompatible with the mark that is being negated and whose joint extension consists in the union of the extensions of the members of this infinite disjunction.[30] An infinite judgement is a judgement that attributes a negative mark to a thing, whereas an affirmative judgement is one that attributes a positive mark to a thing. What matters is not whether one uses a 'non-' predicate (since this is nothing but a linguistic representation), but whether the predicate in which the subject is included is a negative mark that has an infinite 'extension'. It is thus the content of the predicate that determines whether the judgement is affirmative or infinite. In both cases, one is doing the same thing, namely including the subject in the extension of the predicate, yet

[29] Such an account would, moreover, be unable to make sense of the idea that non-P is infinite.

[30] This disjunction is not restricted to predicates with respect to which S can be determined (in which case 'S is not P' would not entail 'S is non-P' when S is not determined with respect to P, even when S is non-empty). This suggestion is misguided. If P can be a differentia of both S and S′, where these can be determined with respect to different predicates, then non-P in the case of S will not be identical to non-P in the case of S′, so that infinitising negation will not be functional. Moreover, this suggestion cannot underwrite the idea that these judgements are infinite. Though the relative complement of P in S is generally larger than P itself, this need not be the case. There can be pairs of predicates that exhaust the possibilities for S, such as 'odd' and 'even' in the case of numbers, in which case the negative predicate and the positive predicate have extensions of the same size and are equally determinate. Finally, in order to specify the relative complement of P in S, one has to make reference to S. The predicate non-P, however, is a differentia that is used to divide the genus and hence cannot make reference to the genus and cannot share any marks in common with S. Instead, the disjunction involves all predicates that are incompatible with P. Though the negative mark itself is not restricted to S, the subject S of the judgement 'S is non-P' restricts non-P to those disjuncts that satisfy the presuppositions of S, so that this judgement includes S in the union of the extensions of all the predicates compatible with S yet incompatible with P. Unlike the relative complement interpretation, this proposal underwrites the idea that it is only the different species, namely SP and Snon-P, that jointly exhaust the sphere of the genus but not the differentiae by themselves.

they differ in terms of whether the predicate picks out a positive or a negative mark.

Double negation elimination also does not apply to copula negation since this type of negation cannot be iterated either. A negative judgement is not the result of applying a negation operator to the copula, where this operator could be iterated and applied arbitrarily many times. Contra Stang, Kant does not conceive of negation as a "one-place truth-function" such that "in a negative judgement what is negated is itself an affirmative judgement" (Stang: 2012, p. 1125). Instead of using a negation operator that yields a negative judgement when applied to an affirmative judgement, Kant is operating with the contrast between inclusion and exclusion. These are two basic ways of connecting subject and predicate, with inclusion giving rise to affirmative judgements and exclusion to negative judgements. A negative judgement is one that excludes the subject from the predicate and exclusion cannot be iterated.

Kant recognises a third type of negation, in addition to copula negation and predicate negation, that likewise does not admit of double negation elimination, namely modal negation: 'false: all/some S is/is not P'. Unlike the other two types of negation, it applies not only to categorical but also to hypothetical and disjunctive judgements. It concerns the judgement as a whole rather than the predicate or the valence of the copula. In particular, it concerns the value of the copula (and more generally the value of the connection between the components, which enables it to encompass hypothetical and disjunctive judgements). Since the modality of a judgement is not part of the content of the judgement (cf. A74/B99-100), one cannot negate a given content to generate a new content that can in turn be negated. Modalities, including negation understood as falsity, cannot be iterated. The question of double negation elimination cannot even be posed in this case.[31]

[31] The distinctiveness of Kant's theory of negation can be fruitfully brought out by considering Stang's suggestion that the principle of complete determination can be synthetic since it is to be understood in terms of infinitising negation '(a is F) or (a is non-F)' and thus distinct from the analytic law of excluded middle 'p or not p', which can be derived from the law of non-contradiction 'not(p and not p)'. This suggestion is off the mark. 1. In Kant's term logic the law of contradiction is not the negation of a proposition and its negation but a constitutive condition on judgements that precludes the predication of a predicate that contradicts the subject, i.e. 'impossible: S is not S'. 2. The 'external' negation of a negative judgement amounts to 'a is not F' rather than 'not(a is F)'. 3. Although a negative judgement can be true when its corresponding infinite judgement is not true, this is not because the "object neither determinately has, nor lacks, the relevant property" (p. 1127), but because S can be contradictory (when concerned with logical truth) or empty (when concerned with material truth) in which case the negative judgement is vacuously true whereas the infinite judgement is false, such that there is no rejection of bivalence. 4. The law of excluded middle is not the disjunction of a proposition and its external negation but the disjunction of a predication and its complementary predication and thus involves 'internal' negation, i.e. 'S is P or S is non-P'. 5. The law of excluded middle, which belongs to transcendental logic, cannot be derived from the law of non-contradiction, which belongs to general logic. 6. The contrast between 'internal' and 'external' negation is misleading in Kant's logic which recognises three forms of negation. Since negation of the copula applies only

## 3.3 Disjunctive judgements

Disjunctive judgements involve opposed judgements that exhaustively and exclusively cover the extension of a concept. The component judgements are coordinated, insofar as they involve complementary predication. The component judgements mutually condition the truth of each other, i.e. the truth of the one grounds the falsity of the others (and vice versa), which is represented by the connection '…or …'.

The opposition and the resulting mutual conditioning amongst the disjuncts is content-based. This is particularly clear in the case of an empirical division. Unlike a logical division which involves only two members, empirical divisions can involve any number of members. Exhaustiveness and exclusiveness of an empirical division cannot be established on logical grounds. Such disjunctive judgements cannot be logically true and hence are not analytic but synthetic judgements. They are materially true when the relevant disjuncts exhaustively and exclusively cover S, not as a matter of logic, but as a matter of fact. In particular, they are materially true when all existing S are divided in accordance with the disjunctive judgement: the non-logical extensions of the various component judgements have to have empty intersections and have to jointly exhaust the non-empty extension of the subject. In that case, they classify as empirical opposites that divide the non-logical extension of S.

Disjunctive judgements based on logical divisions of concepts are likewise content-based. One starts with a concept S and divides it to generate two mutually exclusive possibilities that exhaust the extension of S. Such a logical division uses the positive mark P and its negative complement non-P as differentiae. The opposite of including S in P is including S in non-P rather than excluding it from P. This is because dividing a concept involves forming concepts by including marks, such that one requires negative marks that can be included in concepts and that are the opposites of positive marks. Since such a division involves opposed positive and negative marks, it is only accessible from the perspective of transcendental logic.

From the perspective of general logic, disjunctive judgements are indistinguishable from hypothetical judgements. The coordination involved in a disjunctive judgement is understood in terms of a biconditional, i.e. 'S is P or S is Q' is treated the same way as 'S is P iff S is not Q', which is nothing but a combination of two hypothetical judgements.

Since the disjunctive judgement is formed from two component judgements that include the subject in the extensions of their respective predicates, whereas the biconditional judgement involves one negative judgement, it would seem that these judgements will differ when S is an empty term and hence cannot be treated

to categorical judgements, it is not a good candidate for external negation. The best candidate for an external negation is rather the modal negation 'false' which applies to judgements taken as a whole.

interchangeably. In that case both disjuncts will be false, whereas the biconditional will involve one component judgement that will be false and another that will be true. This impression, however, is misleading since a disjunctive judgement is precisely one in which one and only one disjunct is true. A judgement that involves an empty term simply will not be a disjunctive judgement, even when it is misleadingly stated in a way that suggests that it is a disjunctive judgement (cf. R5964, 18:405-406). Another way to see this is that a logical division divides the extension of S, which presupposes that S is non-empty and has an extension.[32]

Disjunctive judgements are alike to hypothetical judgements, in that they involve a relationship amongst different judgements. Yet, they differ in that disjunctive judgements do not involve a subordination relation whereby the truth of the antecedent grounds the truth of the consequent, but a coordination relation based on the mutual conditioning of the members of the division with the truth of each being a ground of the falsity of the rest and vice versa (cf. B112). Not only is the conditioning mutual rather than asymmetric, it is also not a truth-preserving consequence relation but rather involves a form of classificatory exclusion that relates truths to falsehoods. (The difference is particularly clear in the case of the material truth of the two types of judgements: whereas the material truth of a hypothetical judgement involves a real grounding connection, an empirical division does not involve real grounds but disjuncts that condition each other on the basis of an exclusive and exhaustive division.)

The fact that disjunctive judgements are content-based becomes especially clear when considering judgements with at least three disjuncts. Quarfood has argued against biconditional readings of disjunctive judgements on the grounds that biconditionals involving three arguments are not truth-functionally equivalent to disjunctive judgements in which one and only one of the disjuncts is true (cf. Quarfood: 2013, pp. 311-312). This problem can be resolved by considering the way in which polytomous logical divisions are generated. Whilst the basic case of a disjunctive judgement involves two opposed judgements that are generated by a logical division in terms of P and non-P, this can be extended to polytomous divisions by chaining together different dichotomies. A polytomous logical division is generated by successive dichotomous divisions: "polytomy is a subordinate dichotomy" (R3109, 16:662; also R3269, 16:750).[33]

The differentiae P and non-P in terms of which S is divided can in turn be ex-

[32] It is for this reason that the disjuncts 'S is P' and 'S is non-P' classify as contradictory opposites, despite the fact that both can be false, which can only happen in the case of disparata and not in the case of contradictory opposites. They can both be false only when S is empty, yet this is ruled out when one starts out with S and divides its extension.

[33] Polytomous divisions bring out clearly that disjunctive judgements involve 'non-P' rather than 'not P'. This is because copula negation does not give us a content that can be further divided. Polytomous divisions, however, are only possible if the successive divisions involve concepts whose extensions can in turn be divided into subextensions.

clusively and exhaustively divided in terms of further differentiae Q and non-Q. This allows one, for instance, to generate a disjunctive judgement with three disjuncts: 'S is P or S is non-PQ or S is non-Pnon-Q'. This disjunctive judgement exclusively and exhaustively divides the extension of S into three parts on the basis of two successive logical divisions. This disjunctive judgement is truth-functionally equivalent to the biconditional 'S is P iff S is non-PQ iff S is non-Pnon-Q', given that the predications are not logically independent but are complementary due to being generated by logical divisions that exhaust their respective extensions.

The fact that certain judgements exclusively and exhaustively divide the extension of S and thereby mutually logically ground their truth/falsity is a content-based phenomenon. This is something that is not due to a logical operation that is applied to the component judgements, but due to the content of those judgements. What ensures exclusiveness and exhaustiveness is the nature of the items that are being disjoined, insofar as they involve complementary predications generated by logical divisions. Disjunctive judgements, accordingly, are not to be understood in terms of a logical operation that can be applied to various judgements to form their disjunction. For Kant there is no logical connective 'or' that allows one to disjoin arbitrary judgements. Rather one forms a disjunctive judgement when one mutually subordinates judgements that, due to their contents, exclusively and exhaustively cover the extension of S, such that the truth of each is a ground of the falsity of the others (and vice versa).[34] Put differently, when one mutually subordinates judgements involving different predicates, such that one takes inclusion in one of them to be a ground of exclusion from the others (and vice versa), then it is a matter of the content of these judgements whether one is dealing with one concept that is exclusively and exhaustively divided, such that one is forming a disjunctive judgement, or whether one is merely forming a number of separate hypothetical judgements.

The importance of operating with infinite judgements, rather than negative judgements, can also be brought out by considering the law of excluded middle, which is the principle of disjunctive judgements. This law is not to be understood in terms of the truth or falsity of one and the same judgement 'true: S is P' or 'false: S is P'. Instead, it requires that of two contradictorily opposed predicates exactly one belongs to the subject. That is, the two possibilities are to be understood in terms of predicating positive and negative marks: 'S is P' or 'S is non-P', which ensures that this law can only be stated in transcendental logic and is not derivable from the law of non-contradiction, which belongs to general logic. The opposition between positive and negative marks allows one to infer the truth of one disjunct from the falsity of the other (and vice versa), i.e. if the subject is excluded from one predicate then it has to be included in the other (and vice versa)

[34] In the case of the material truth of disjunctive judgements based on empirical divisions, it is the contents of the judgements together with the contingent state of the world that explains the mutual conditioning of the disjuncts.

– "exclusi medii: where through the falsity of the opposite truth is demonstrated" (R2178, 16:260).

| | |
|---|---|
| 1. | S is P or S is non-P |
| 2. | S is P |
| ∴ | S is not non-P |

The major premise of a modus ponendo tollens consists of a disjunctive judgement that identifies the two possibilities. Each disjunct is entertained problematically, whilst the disjunction is judged assertorically. The minor premise includes S in the positive mark P. The conclusion then excludes S from the other disjunct, namely from the negative mark non-P.

This inference only works for 'non-P' but not for 'not P', which means that the law of excluded middle cannot be understood in terms of 'S is P' or 'S is not P'. This is because if 'S is P' is true, i.e. if S is included in P, then there is nothing from which S is to be excluded. One cannot exclude it from 'not P', since 'not P', unlike 'non-P', does not have an extension. 'S is not P' is already a negative judgement that involves exclusion and exclusion cannot be iterated. One can only assert its opposite, yet that is simply 'S is P'. In that case one would have a degenerate disjunctive syllogism with the conclusion being a mere restatement of the minor premise and the major premise not playing any role.[35]

| | |
|---|---|
| 1. | S is P or S is not P |
| 2. | S is P |
| ∴ | S is P |

It might be suggested that one could infer the falsity of 'S is not P'.

| | |
|---|---|
| 1. | true: S is P or S is not P |
| 2. | true: S is P |
| ∴ | false: S is not P |

However, this does not work either, since one cannot infer 'false: S is not P', given that the conclusions of syllogisms are always judged apodictically (cf. JL 9:122), which is incompatible with the logical modality 'false'.[36]

The fact that it is only from the perspective of transcendental logic that one can understand disjunctive judgements can be further illustrated by considering the quantity of the component judgements. Disjunctive judgements are complex judgements that are formed by synthesising component judgements. Disjunctive syllogisms are only possible if these component judgements can be detached and be self-standing. For instance, in a disjunctive syllogism in which one infers from

[35] From the perspective of general logic, the disjunction would collapse into the trivial biconditional 'S is P iff S is P'.

[36] In the case of modus tollendo ponens the conclusion would be judged apodictically, yet the minor premise 'false: S is not P' would not be judged assertorically.

the truth of one of the disjuncts to the falsity of the others, the minor premise will be a categorical judgement that corresponds to one of the disjuncts of the disjunctive judgement. Whilst they differ in terms of their modality, insofar as the minor premise is judged assertorically whereas the disjunct of the major premise is judged problematically, they have to be identical in terms of quantity, quality and relation. Otherwise, they cannot function as premises and conclusions of disjunctive syllogisms. This implies that the categorical component judgements have to have a quantity. Specifying the quantity of the component judgements, however, is difficult.

Opposed cognitions can straightforwardly be accounted for in terms of the square of opposition: universal affirmative judgements will be opposed by particular negative judgements and likewise for universal negative and particular affirmative judgements. However, one is doing something different when dividing S in terms of P and non-P (cf. R3096, 16:657-658).[37] In a concept division, one takes the intersections of S and P as well as of S and non-P to form two new concepts. Rather than judging that the extension of S is included in that of P (in which case one could straightforwardly specify whether it is the entire extension or only a part thereof), one combines P with S to form a new concept. Whilst the generated concepts can be completely subsumed under P, i.e. 'all SP are P' will be entailed by this division, the disjunctive judgement concerns S itself. Here, one can only establish the particular judgements 'some S is P' and 'some S is non-P'. These, however, are not opposed and do not generate a disjunction.

What one rather seems to be saying is that the things that are S are either P or non-P. This, however, is not a disjunction of judgements, but a judgement with a disjunctive predicate. We would then be operating with a categorical judgement with a complex disjunctive predicate rather than with a complex judgement that is constructed out of categorical judgements. Disjunctive judgements, however,

[37] Occasionally, Kant characterises disjunctive judgements so that they are not restricted to the division of concepts but also encompass disjunctions based on opposed cognitions. "Every disjunction is a division of the *Sphaera* of cognition: either of a given concept or of truth as such." (R3097, 16:659) Similarly, R3098 distinguishes "disjunction of the division of a concept, otherwise there is disjunction of mere opposition" (16:659). In the case of opposed cognitions, one is not operating with contradictorily opposed predicates but with contradictorily opposed judgements. Here, one can also infer the truth/falsity of a judgement from the falsity/truth of its opposite, since exactly one of two contradictorily opposed judgements is true. One does so, however, via an immediate inference of the understanding per judicia contradictorie opposita (cf. JL 9:116-117), not via a disjunctive syllogism which is a mediate inference of reason. (Although disjunctive syllogisms, strictly speaking, do not classify as mediate inferences of reason on the same grounds that hypothetical syllogisms fail to be mediate inferences (cf. JL 9:129; R3264-3265, 16:747), since they do not involve a middle term but contain the disjunction/consequence in the major premise, they nevertheless involve one judgement that is subsumed under a rule and taken to be the condition of the other and thus require both a major and a minor premise, such that, unlike in the case of opposed cognitions, one cannot get directly from one judgement to the other.) Relatedly, the opposition of opposed cognitions is not based on content but on form and is accessible in general logic and not only in transcendental logic.

have categorical judgements as their components. As Kant notes, "categorical judgements make up the matter of the remaining ones" (R3046, 16:631). Like hypothetical judgements, they involve a conditioning relation amongst different judgements. One needs to establish a mutual conditioning relation at the level of judgements, not at the level of the disjuncts of a disjunctive predicate, given that "the disjunctive judgement contains a relation of two or more propositions with respect to one another" (A73/B98-99).

The problem now is that there are no suitably opposed judgements when S is divided on the basis of both P and non-P, as long as one operates with the extension of S. The universal judgements 'all S are P' and 'all S are non-P' are disparata and in this case both false. The particular judgements 'some S is P' and 'some S is non-P' are subcontraries that do not oppose each other and in this case both true. This problem can be resolved by operating with singular judgements. The requisite opposition can be found at the level of the individual members of the extension of S. The disjunctive judgement 'the S is P or the S is non-P' is composed of singular categorical judgements that oppose each other. Correspondingly, disjunctive syllogisms, in which one infers the truth of one disjunct from the falsity of the others (and vice versa), have singular judgements as their minor premise and conclusion and the disjunctive judgement that constitutes the major premise is composed of singular judgements.

Moreover, only singular judgements can make sense of the multiple ways in which disjunctive judgements can be realised. A concept division represents the range of possibilities for the things falling under the concept. For each thing, exactly one of the disjuncts is true, yet which disjunct it is that is true will vary. Different things will realise different possibilities, i.e. one will be P whereas another one will be non-P. Correspondingly, one disjunctive syllogism will issue in the falsity of non-P, whereas another will issue in the falsity of P. The requisite variation can only be found when operating with singular judgements, insofar as different members of the extension of S can realise the different possibilities. The truth of 'some S is P' as well as the falsity of 'all S are P', by contrast, are invariant.

The fact that disjunctive judgements based on logical divisions involve both singular and infinite judgements implies that transcendental logic is required to distinguish them both because of the singularity of their subjects and because of the mutual conditioning resulting from complementary predication. The requisite opposition is content-based, both with regard to the quantity and the quality of the component judgements.[38]

[38] It also means that, like in the other tables, the third forms under each heading go together.

# References


[1] Bader, R. M. Kantian Metaontology. In *Routledge Handbook of Metametaphysics*, R. Bliss and J. Miller, Eds. Routledge, 2021, pp. 23–31.

[2] Jesiołkiewicz, J. *Das unendliche Urteil „Die Seele ist nichtsterblich" (anima est nonmortalis) – Ein Versuch, das bekannte Lehrstück Immanuel Kants aus der ›Kritik der reinen Vernunft‹ (A 71/B 96–A 72/B 97) vor dem Hintergrund der Logik des 18. Jahrhunderts zu beleuchten*. PhD thesis, Ludwig-Maximilians-Universität München, 2020.

[3] Kant, I. *Kants gesammelte Schriften*. Reimer/de Gruyter, 1900.

[4] Kiesewetter, I. G. C. *Versuch einer faßlichen Darstellung der wichtigsten Wahrheiten der neuern Philosophie für Uneingeweihte*, 2nd ed. Wilhelm Oehmige, 1798.

[5] Korte, T., and Repo, A. The problem of singular judgments in Kant. *History of Philosophy Quarterly 28*, 4 (2011), 389–406.

[6] Land, T. Intuition and judgment – How not to think about the singularity of intuition (and the generality of concepts) in Kant. In *Kant und Philosophie in weltbürgerlicher Absicht – Akten des XI. Internationalien Kant-Kongresses*, S. Bacin, Ed., vol. 2. de Gruyter, 2013, pp. 221–231.

[7] Longuenesse, B. *Kant and the Capacity to Judge*. Princeton University Press, 1998.

[8] Meier, G. F. *Auszug aus der Vernunftlehre*. Johann Justinus Gebauer, 1752.

[9] Proops, I. *The Fiery Test of Critique – A Reading of Kant's Dialectic*. Oxford University Press, 2021.

[10] Quarfood, M. Interpretations of Kantian disjunctive judgment in propositional logic. In *Kant und Philosophie in weltbürgerlicher Absicht – Akten des XI. Internationalien Kant-Kongresses*, S. Bacin, Ed., vol. 2. de Gruyter, 2013, pp. 307–319.

[11] Rosefeldt, T. *Das logische Ich – Kant über den Gehalt des Begriffes von sich selbst*. Philo, 2000.

[12] Stang, N. Kant on complete determination and infinite judgement. *British Journal for the History of Philosophy 20*, 6 (2012), 1117–1139.

[13] Thompson, M. Singular terms and intuitions in Kant's epistemology. *The Review of Metaphysics 26*, 2 (1972), 314–343.