1. Metric spaces and isometries

Pseudo-metrics and metrics, normed spaces, classical sequence spaces; isometries of normed spaces, theorem of Mazur-Ulam, example of a non-affine isometric embedding; Lipschitz maps and examples

2. Topological notions, completeness

The topology of a metric space, metrization theorem of Nagata-Smirnov; compactness, completeness, extension of uniformly continuous maps defined on a dense subset; $C_b(X, X')$ complete if X' is; Kuratowski embedding, completion of a metric space, Banach fixed point theorem

3. Baire's theorem

Baire spaces, Baire category theorem, meagre and co-meagre subsets; applications of Baire's theorem: nowhere monotone functions, nowhere differentiable functions, Baire classes, discontinuity sets for the first class of Baire, examples; open mapping theorem for Banach spaces, application to linear ODEs

4. Compactness

Separability, compactness, precompact = totally bounded, characterization in terms of sequences, Lebesgue number; example French railway space; $\varepsilon-$separated subsets; isometries of a compact metric space: distance preserving implies bijective, 1-lipschitz and surjective implies isometry, expanding implies isometry; compactness in $C(X, Y)$, Arzela-Ascoli theorem

5. More on compactness, embeddings

Schauder bases, the basis problem for separable Banach spaces, Mazur's goose, examples, the basis constant, compactness criterion in Banach spaces with bases; every separable metric space embeds topologically into the Hilbert cube; compactness criterion in $l_\infty(S)$; Frechet-embedding, uniform precompactness, Gromov-embedding into compact subsets of $l_\infty(\mathbb{N})$

6. Dimensions

Covering dimension $\mathrm{covdim}(X)$, embedding theorem of Menger-Nöbeling and Hurewicz, topological embeddings form a dense G_δ-set in $C(X, I_{2n+1})$; inductive dimension, equal to $\mathrm{covdim}(X)$ if X separable metric; Hausdorff measure as a metric outer measure, Hausdorff dimension, $\mathrm{inddim}(X) \leq \mathrm{hausdim}(X)$, Marczewski's theorem, Hausdorff dimension of Cantor sets C_λ, hausdim is not a topological invariant, behaviour under Hölder continuous maps; upper and lower box dimension

7. Hausdorff distance

Definition of d_H, $\mathrm{Clos}(X)$ is complete or precompact if X is, upper closed limits, Hausdorff limits and topological limits; application to iterated function schemes, Banach fixed point theorem applied to Hausdorff distance, formula for $\mathrm{hausdim}(F)$; alternative description of $d_H(A, B)$ using d_A and d_B, Busemann distance

8. Gromov-Hausdorff distance

Definition of d_{GH} via embeddings and d_H, Lipschitz-close implies GH-close, d_{GH} is a metric on isometry classes of compact spaces, not so for proper spaces; description using embeddings into l_∞, distortion of correspondences and ε-isometries; properties inherited by GH-limits, n-point conditions, δ-hyperbolicity

9. Compactness for d_{GH}, doubling

Packing numbers and covering numbers, characterizations of precompact subsets of Gromov-Hausdorff space $\mathbf{M}$, completeness of $\mathbf{M}$; application to Riemannian geometry: Bishop-Gromov volume comparison theorem, covering lemma, Gromov's compactness theorem for Ricci curvature; doubling metric spaces, doubling measures, doubling measure implies doubling metric, complete doubling metric implies existence of a doubling measure μ, Dynkin conjecture, construction of μ for finite spaces; pointed GH-convergence, compactness theorem for pointed GH-convergence

10. Outlook: Alexandrov spaces

Length spaces, geodesic spaces, curvature bounds via triangle comparison, properties of Alexandrov spaces