

## ABSTRACTS

### **Manuel Amann: An “all-round” view to positive quaternion Kähler geometry**

A positive quaternion Kähler manifold is a Riemannian manifold with holonomy contained in  $Sp(n)Sp(1)$  and positive scalar curvature. Conjecturally, such a manifold is a symmetric space. Recently, positive quaternion Kähler geometry has become a pretty popular field of study. In this talk I shall comment on known results and I shall discuss further properties these manifolds share with symmetric spaces. Presented results will vary from confirmations of the conjecture in special cases to an outline of characteristics rather algebraically topological in nature.

### **Roger Bielawski: Generalised and asymptotic symmetries of hyperkaehler manifolds**

A generalised symmetry of a hyperkaehler manifold is a higher rank Killing spinor. On many interesting hyperkaehler manifolds, such as different moduli spaces of topological solitons, certain quiver varieties, etc, the “group” of generalised symmetries has maximal rank. Moreover, these generalised symmetries become asymptotically genuine isometries. I shall review and extend constructions of such metrics (generalised Legendre transform, foliations of generalised hyperkaehler manifolds) and compare them to toric hyperkaehler manifolds. I shall also discuss their topological and geometric asymptotics.

### **Andrew Dancer: Ricci solitons with large symmetry group**

We produce new examples of Ricci solitons by analysing reductions of the equations to a system of ordinary differential equations. Some of the new examples are non-Kähler.

**Fuquan Fang: 5-manifold with positively pinched sectional curvature (joint work with X. Rong)**

Let  $M$  be a closed 5-manifold of pinched curvature  $0 < \delta \leq \sec_M \leq 1$ . In this talk we will brief the main idea to prove some topological rigidity theorems about positively curved 5-manifolds, e.g., that  $M$  is homeomorphic to a spherical space form if  $M$  satisfies one of the following conditions: (i)  $\delta = 1/4$  and the fundamental group is a non-cyclic group of order  $\geq C$ , a constant. (ii) The center of the fundamental group has index  $\geq w(\delta)$ , a constant depending on  $\delta$ . (iii) The volume is less than  $\epsilon(\delta)$  and the fundamental group  $\pi_1(M)$  has a center of index at least  $w$ , a universal constant, and  $\pi_1(M)$  is either isomorphic to a spherical 5-space group or has an odd order.

**Bernhard Hanke: The stable rank of symmetry of  $S^{n_1} \times \dots \times S^{n_k}$**

A well known conjecture in the theory of transformation groups states that if  $p$  is a prime and  $(\mathbb{Z}/p)^r$  acts freely on a product of  $k$  spheres, then  $r$  is less than or equal to  $k$ . We prove this assertion if  $p$  is large compared to the dimension of the sphere product. The argument builds on tame homotopy theory for non simply connected spaces.

**Ewa Kozłowska-Walania: Symmetries of compact Riemann surfaces**

It is known that every compact Riemann surface corresponds to a projective irreducible smooth complex algebraic curve and it admits an antiholomorphic involution (symmetry in short) if and only if the corresponding curve has a real form. The goal of the talk is to present the quantitative and qualitative results concerning symmetries of compact Riemann surfaces. First we give some results on the maximal number of non-conjugate symmetries of a Riemann surface of genus  $g$ , which equals the number of non-isomorphic real forms of a complex algebraic curve of genus  $g$ . Then we focus on the topology of the set of points fixed by a symmetry as it is homeomorphic to the smooth projective model of the respective real form and by the classical result of Harnack consists of at most  $g + 1$  disjoint simple closed curves called ovals. We give some bounds for the total number of ovals of two symmetries in terms of the genus, the order of their product and the number of its fixed points. As a corollary we obtain the lower bound for  $g$  that guarantees commutativity of

two symmetries and we show that it is minimal, with unique exception in any genus. Finally, for a pair of commuting symmetries with given separabilities we find ranges, for the hyperellipticity degree of their product and we present the bound for the total number of ovals of  $k$  non-conjugate symmetries of a Riemann surface of genus  $g$ .

### **Daniele Otera: The topology of balls in metric complexes**

The notion of quasi-simple filtration (qsf) has been introduced by Brick in the 90's, and amounts to finding an exhaustion of the Cayley 2-complex of a group “approximable” by finite simply connected complexes.

The present talk consider the relations between the qsf and other tameness conditions for groups and manifolds. Moreover we will consider a closely related property for metric complexes and observe what happens if one require the exhaustion to be the one by metric balls.

### **Volker Puppe: Involutions on 3-manifolds and self-dual, binary codes (joint work with Matthias Kreck)**

We study a correspondence between orientation reversing involutions on compact 3-manifolds with only isolated fixed points and self-dual, binary codes. We show in particular that every such code can be obtained from such an involution.

### **Frank Reidegeld: $Spin(7)$ -manifolds of cohomogeneity one**

At the beginning of the talk, a classification result on the possible principal orbits of cohomogeneity-one  $Spin(7)$ -structures will be presented. The case where the principal orbit is an Aloff-Wallach space we explore in detail. We discuss some of the problems which are related to the equations for the holonomy reduction and give examples of old and new cohomogeneity-one metrics whose holonomy is contained in  $Spin(7)$ . Some of the other principal orbits will be shortly considered, too. As a by-product, the local existence of certain cohomogeneity-one Einstein metrics can be proven.

### **Lorenz Schwachhöfer: Manifolds with nonnegative curvature**

Most of the known examples of closed manifolds with nonnegative sectional curvature admit an isometric group action of cohomogeneity one. On the other hand, not all cohomogeneity one manifolds admit invariant metrics of nonnegative curvature. In this talk, we shall discuss existence and non-existence results for nonnegatively curved metrics under the additional hypothesis that there is a totally geodesic principle orbit.

### **Andras Szenes: The cohomology of representation varieties**

We present recent progress in the study of the cohomology of moduli spaces of flat non-unitary connections over Riemann surfaces. Using hyperkahler geometry and equivariant integration, we arrive at a conjectured formula for the cohomology ring, which involves a natural deformation of the determinantal hyperplane arrangement.

### **Ewa Tyszkowska: Symmetries of elliptic-hyperelliptic Riemann surfaces**

Exceptional points in the moduli space of compact Riemann surfaces are unique surface classes whose full group of conformal automorphisms acts with a triangular signature. Their defining equations as algebraic curves have coefficients in a number field. A surface admitting a conformal involution with quotient being an elliptic curve is called elliptic-hyperelliptic. A symmetry of a Riemann surface is an antiholomorphic involution; a surface is symmetric if it admits a symmetry. A set of fixed points of a symmetry of a surface of genus  $g$  consists of  $k$  disjoint Jordan curves called ovals, where  $0 \leq k \leq g + 1$ .

We determine, up to topological conjugacy, the full group of conformal and anticonformal automorphisms of a symmetric exceptional point in the elliptic-hyperelliptic locus and find the number of ovals of any symmetry of such surface. We show that while the elliptic-hyperelliptic locus can contain an arbitrary large number of exceptional points, no more than four are symmetric.

## **Gregor Weingart: About the Spectrum of the Dirac Operator on Normal Homogeneous Spaces**

In contrast to the special case of Riemannian symmetric spaces there seems to be no feasible algorithm for the calculation of the spectrum of the Dirac operator on spinors or the Laplace operator on differential forms. The standard approach to this problem uses a deformation of the operator in question to embed it into a larger algebra of differential operators, in particular the so called “cubic Dirac operator” arises naturally this way. We will discuss this standard approach and present the ensuing calculations for a couple of normal homogeneous spaces in dimensions 6 and 7. The results of these calculations suggest that despite all the difficulties the spectrum of the Dirac and Laplace operators on normal homogeneous spaces may have a simpler description than could be expected.

## **Michael Wiemeler: Torus manifolds with non-abelian symmetries**

Since the 1970s toric varieties were studied by algebraic geometers. This study led to many applications in algebraic geometry and other areas of mathematics.

In the 1990s M.Davis and T.Januszkiewicz and M.Masuda introduced two generalizations of non-singular toric varieties, quasitoric manifolds and torus manifolds. A torus manifold is a  $2n$ -dimensional manifold on which a  $n$ -dimensional torus acts almost effectively such that  $M^T \neq \emptyset$ . A quasitoric manifold is a torus manifold whose orbit space  $M/T$  is a simple convex polytope.

In my talk I will discuss classification results for torus manifolds and quasitoric manifolds with an action of a compact connected non-abelian Lie-group which extends the torus action.

## **Burkhard Wilking: Nonnegatively curved manifolds with symmetries**

The structure of positively curved manifolds with symmetry has enjoyed quite a bit of attention in recent years and a lot of progress has been made. The structure of nonnegatively curved manifolds with symmetry is not as well understood. We sketch a few results in this context.