WORKSHOP ON GEOMETRY AND TOPOLOGY ABSTRACTS

Université de Fribourg, May 17-19, 2007

Daniel Maerten: Penrose-like inequality for maximal asymptotically flat spin initial data sets

We prove a Penrose-like inequality for the mass of a large class of constant mean curvature (CMC) asymptotically flat n-dimensional spin manifolds which satisfy the dominant energy condition and have a future converging, or past converging compact and connected boundary of non-positive mean curvature and of positive Yamabe invariant. We prove that for every n greater than 3, the mass is bounded from below by an expression involving the norm of the linear momentum, the volume of the boundary, dimensionless geometric constants and some normalized Sobolev ratio.

The main interest of this result is to give an idea of what a plausible conjecture for the Penrose inequality with extrinsic curvature could be.

Ernst Heintze: Kac-Moody algebras and symmetric spaces

We discuss infinite dimensional symmetric spaces whose isometry group is an affine Kac-Moody group.

Bernd Siebert: Affine manifolds and string geometry

Recent progress in our understanding of the mirror phenomenon indicates that real manifolds with an affine structure play a central role in string geometry. In fact, it now seems that most of the known mathematical structures arising from string theory can be expressed in terms of geometry on some affine manifold.

In the talk I will argue in favor of this claim by commenting on recent joint work with Mark Gross on the reconstruction problem. This work in particular links string geometry and affine geometry in a much more explicit and useful fashion than any previous method.

Michelle Bucher-Karlsson: Simplicial volume of products of surfaces

The simplicial volume of a closed, oriented manifold M is a topological invariant introduced by Gromov in the early 1980's which roughly speaking measures how complicated it is to represent the fundamental cycle of the manifold M by real singular cycles. If M is a Riemannian manifold, then the simplicial volume of M is proportional to the volume of M by a constant (possibly infinity) depending only on the universal cover of M. This shows that, within classes of manifolds isometrically covered by a given Riemannian manifolds for which the proportionality constant is finite, the volume is a topological invariant.

For hyperbolic manifolds, the proportionality constant is equal to the maximum volume of ideal geodesic simplices in the hyperbolic space. I will show how to compute the proportionality constant for products of two copies of the hyperbolic plane, and in particular the simplicial volume of products of hyperbolic surfaces. This gives the first exact value of a nonvanishing simplicial volume for a manifold of nonconstant curvature.

Matthias Franz: Cohomology of toric varieties and their real parts

Toric varieties are certain complex algebraic varieties equipped with an action of a torus $(S^1)^n$. They provide a link between geometry and combinatorics because they can be defined in terms of polytopes and other convex geometric objects. Like all varieties defined over the reals, they come with a globally defined complex conjugation. Its fixed point set is the corresponding real toric variety, on which a "2-torus" $(\mathbf{Z}/2\mathbf{Z})^n$ acts.

In my talk I will describe both classical results as well as recent developments about the relation between the cohomology of a toric variety and that of its real part.

François Guéritaud: Convex cores of quasifuchsian groups

We study a continuous family of hyperbolic 3-manifolds, called quasifuchsian (homeomorphic to the product of a punctured torus with the real line). The convex core of such a space carries so-called pleating laminations, which are topological/combinatorial information. We will show how this information completely determines a purely metric invariant, the Delaunay triangulation. Appealing drawings will be shown, in the spirit of the book "Indra's Pearls".

Jean-Claude Hausmann: The topology and geometry of polygon spaces

The study of polygon spaces in R^d has been developped during the last two decades. They occur in connection with statistical shape theory and robotics. For $d = 3$, they became also a chapter of Hamiltonian geometry, as a good source of examples, closely related to toric manifolds. This talk will be a survey of these various aspects of polygon spaces.

Kathryn Hess: Free loop spaces and closed geodesics

Gromoll and Meyer showed in 1969 that a closed, compact manifold M of dimension at least two admits infinitely many distinct, closed geodesics with respect to any Riemannian metric if there is a field $\mathbf{k}$ such that the Betti numbers $\{\dim H^i(M^{S^1};\mathbf{k})\}_i$ of the free loop space on M are unbounded. In 1976 Sullivan and Vigué showed that if M is simply connected and its rational cohomology is non-monogenic, then the Betti numbers $\{\dim H^i(M^{S^1};\mathbf{Q})\}_i$ are unbounded and thus that M admits infinitely many distinct, closed geodesics.

In this talk I will outline a program (almost completed) for proving a generalization of the theorem of Sullivan and Vigué to coefficients in any field. This is joint work with N. Dupont and J. Scott.