

# Proseminar SA 2016

## – Expander graphs –

Responsible: Dr. Corina Ciobotaru

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Tuesdays 15:15 - 17:00

Room: auditoire 2.52, Bâtiment de Physiques

Languages: Français, Deutsch, English

Website: <http://homeweb.unifr.ch/CiobotaC/Pub/Proseminar/>

The aim of this Proseminar is to read the book by Giuliana Davidoff, Peter Sarnak and Alain Valette entitled “Elementary Number Theory, Group Theory and Ramanujan Graphs” [DSV03]. The book is meant for undergraduate students and is self-contained, the prerequisites being only linear algebra, elementary algebra, analysis and combinatorics.

Roughly speaking, **expander graphs** are highly connected sparse finite graphs, our goal being to study and understand this notion and to give explicit constructions. Expanders play an important role in computer science as basic building blocks for network constructions, error correcting codes, algorithms and more. They also play a significant role in pure mathematics like number theory, group theory, geometry and more.

**This Proseminar is compulsory for third year students in Mathematics.**

## References

- [DSV03] Giuliana Davidoff, Peter Sarnak, and Alain Valette, *Elementary Number Theory, Group Theory and Ramanujan Graphs*, Cambridge University Press, Cambridge, 2003. ↑1
- [Kow16] Emmanuel Kowalski, *Expander graphs* (2016). ETH Zurich, Spring Semester 2016 Version of February 24, 2016. ↑
- [Lub94] Alexander Lubotzky, *Discrete Groups, Expanding Graphs and Invariant Measures*, Springer, Birkhäuser Verlag, 1994. ↑
- [Ser80] Jean-Pierre Serre, *Trees*, Springer Monographs in Mathematics, Springer-Verlag, Berlin, 1980. Translated from the French original by John Stillwell. ↑
- [Tit70] Jacques Tits, *Sur le groupe des automorphismes d’un arbre*, Essays on topology and related topics (Mémoires dédiés à Georges de Rham), Springer, New York, 1970, pp. 188–211 (French). ↑

# Planning of the Talks

1. Sections 1.1 and 4.1: *The adjacency matrix of a graph and its spectrum, Cayley graphs*. **All the exercises at the end of the sections are compulsory**. Difficulty: easy. STAUDT Denis <denis.staudt@unifr.ch>
2. Section 1.2: *Inequalities of the spectral gap*. **All the exercises at the end of the section are compulsory**. Difficulty: medium. ZIMMERMANN Severine <severine.zimmermann@unifr.ch>
3. - 4. (For two students, working together)  
Sections 1.3 and 1.4: *Asymptotic behaviour of eigenvalues in families of expanders, Proof of the asymptotic behaviour*. **Some of the exercises at the end of the sections are compulsory**. Difficulty: medium to difficult. SCHUHMACHER Rahel <rahel.schuhmacher@unifr.ch> Christian Vogt <christian.vogt@unifr.ch>
5. Sections 1.5 and 1.6: *Independence number and chromatic number, Large girth and large chromatic number*. **All the exercises at the end of the sections are compulsory**. Difficulty: easy to medium. GRAND-GUILLAUME-PERRENOUD Davina <davina.grand-guillaume-perrenoud@unifr.ch>
6. Section 2.1: *Sums of two squares*. **All the exercises at the end of the section are compulsory**. Difficulty: easy. COPPEX Jeannine <jeannine.coppex@unifr.ch>
7. Sections 2.2 and 2.3: *Quadratic reciprocity, Sums of 4 squares*. **All the exercises at the end of the sections are compulsory**. Difficulty: easy. SCHALLER Vinzenz Theodor <vinzenztheodor.schaller@unifr.ch>
8. Sections 2.4 and 2.5: *Quaternions, The arithmetic of integer quaternions*. **All the exercises at the end of the sections are compulsory**. Difficulty: easy to medium. Julie Raniolo <julie.raniolo@bluewin.ch>
9. Sections 3.1 and 3.2: *Some finite groups, Simplicity*. **All the exercises at the end of the sections are compulsory**. Difficulty: medium. FRONTINI Magali <magali.frontini@unifr.ch>
10. Section 3.3: *Structure of subgroups*. **Some of the exercises at the end of the section are compulsory**. Difficulty: medium. PRIVITERA Aurelio <aurelio.privitera@unifr.ch>
11. - 12. (For two students, working together)  
Section 3.4 and 3.5.: *Representation theory of finite groups, Degrees of representations of  $\mathrm{PSL}_2(q)$* . **Some of the exercises at the end of the sections are compulsory**. Difficulty: medium to difficult. SKOPA Eugene <eugene.skopa@unifr.ch>  
**waiting for a partner**

13. Section 4.2 and 4.3: *Construction of  $X^{p,q}$ , Girth and connectedness*. **Some of the exercises at the end of the sections are compulsory**. Difficulty: medium to difficult. Clelia LIEBERMANN <clelia.liebermann@gmail.com>
14. Section 4.4: *Spectral estimates*. **All the exercises at the end of the section are compulsory**. Difficulty: medium to difficult. ROSSET Julien <julien.rosset@unifr.ch>
15. Appendix A: *4-regular graphs with large girth*. **All the exercises at the end of the section are compulsory**. Difficulty: medium to difficult.

More talks will be available soon, depending on the number of students.

## Requirements to pass the Proseminar

You have to give a blackboard talk of 90 minutes (without using your notes, do not copy from your notes on the blackboard!), to answer questions from the audience and to write a manuscript of 5 pages containing a summary of your talk and the solutions of some of the exercises at the end of the corresponding section of the book. The manuscript can be ready after you give the talk. When writing the manuscript, please do not copy directly from the book. To write a good manuscript you have to use your own words and your own interpretation/structure of the topic. One way to do this is to try to write from your memory the statements of the theorems/propositions/lemmas and their proofs. This is a good way to see if you well understand what are you reading.

To prepare a good talk, give examples after every definition/theorem and use your own interpretation/structure than the one in the book. To be able to fit the talk in 90 minutes, you should repeat your talk at home 3-4 times (without any notes!!). Eventually, you can ask your colleagues to listen to your talk and to give you a feedback.